

Interesting Simulation (Maxima)

7.11.2024
Sohun

0 – 1 Introduction

The French-born philosopher Rene-Descartes (1596-1650) is known as the "father of modern philosophy". He is the man who coined the famous proposition, "I think, therefore I am". Descartes was also a mathematician, investing concepts such as coordinates and calculus.

Descartes linked figures and equations (analytical geometry).

For example, in high school mathematics, students learn that on an xy coordinate plane, a parabola (figure) is the figure respected by $y=ax^2+bx+c$ (equation). Similarly, students learn that a straight line (figure) is the figure respected by $y=ax+b$ (equation), and a circle (figure) is the figure represented by $x^2+y^2=c^2$ (equation), too.

Furthermore, students learn that in xyz coordinate space, a sphere (figure) is the figure represented by $x^2+y^2+z^2=c^2$ (equation).

So, what shape does the equation $(x^2 + y^2)^4 - (x^2 - y^2)^2 = 0$ represent?

The free software "Maxima" answers this question. The answer is shown in the diagram on the right. It is a four-leaf clover inscribed in a circle of radius 1.

I entered the equation $x^2 + y^2 = 1$ for a circle of radius 1 and equation $(x^2 + y^2)^4 - (x^2 - y^2)^2 = 0$ into "Maxima" and drew it.

By the way, the figure represented by equation $x^2 + y^2 + z^2 = 1$ is a sphere with a radius of 1. So, what shape is represented by equation $x^2 + y^2 - z^2 = 1$, which is the equation for a sphere with the + changed to a -?

What is the shape represented by equation $x^2 - y^2 - z^2 = 1$?

"Maxima" can answer these questions.

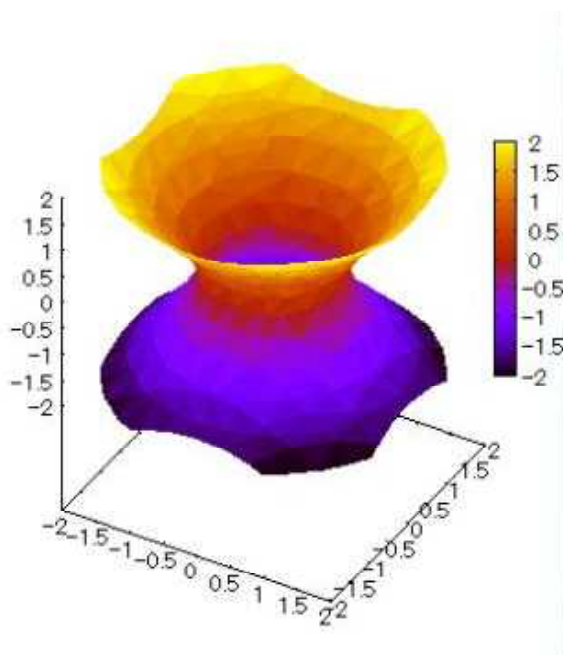
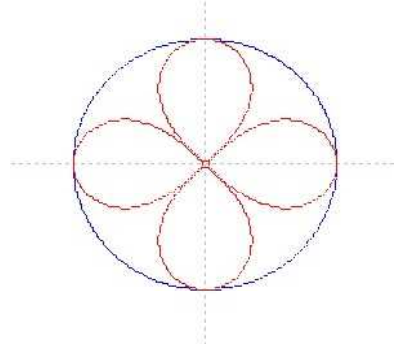
The figure below is what is drawn when equation $x^2 + y^2 - z^2 = 1$ and equation $x^2 - y^2 - z^2 = 1$ are entered into "Maxima" respectively.

In this way, I entered various equations into "Maxima" and had it draw them.

In addition, the shapes drawn by "Maxima" can be observed from various angles.

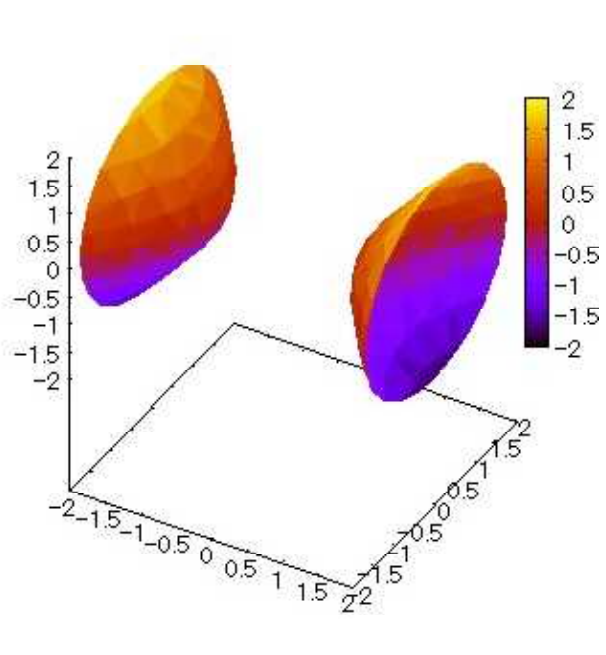


Rene · Descartes



$$x^2 + y^2 - z^2 = 1$$

$$(-2 \leq x \leq 2) (-2 \leq y \leq 2) (-2 \leq z \leq 2)$$



$$x^2 - y^2 - z^2 = 1$$

$$(-2 \leq x \leq 2) (-2 \leq y \leq 2) (-2 \leq z \leq 2)$$

Interesting Simulation (Maxima)

6.26.2024
Sohun

1 – 1 3D Implicit Functions

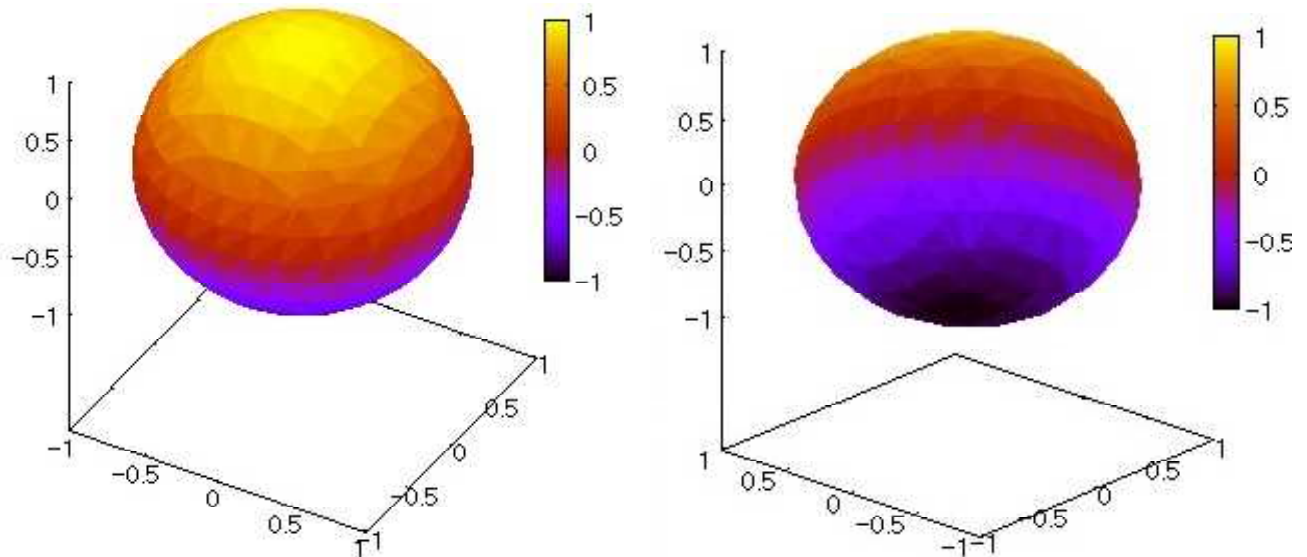
(1) Function Expression

$$x^2 + y^2 + z^2 = 1 \quad (-1 \leq x \leq 1) \quad (-1 \leq y \leq 1) \quad (-1 \leq z \leq 1)$$

(2) Input Expression

```
draw3d(enhanced3d=true,implicit(x^2+y^2+z^2=1,x,-1,1,y,-1,1,z,-1,1));
```

(3) Rendering Result [Sphere]



1 – 2 3D Implicit Functions

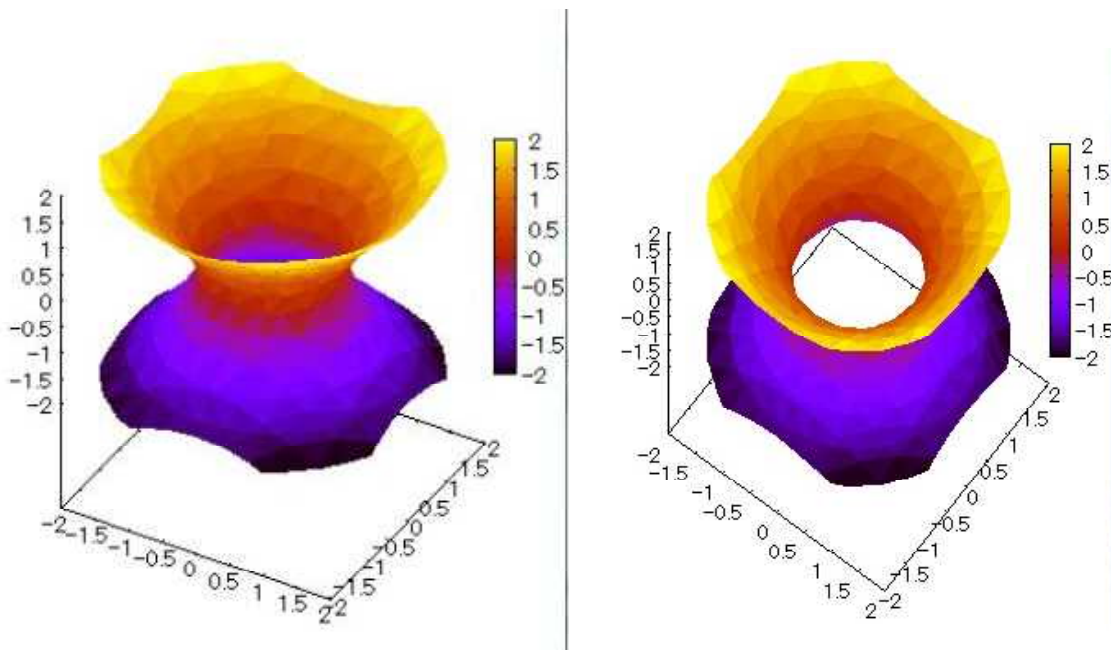
(1) Function Expression

$$x^2 + y^2 - z^2 = 1 \quad (-2 \leq x \leq 2) \quad (-2 \leq y \leq 2) \quad (-2 \leq z \leq 2)$$

(2) Input Expression

```
draw3d(enhanced3d=true,implicit(x^2+y^2-z^2=1,x,-2,2,y,-2,2,z,-2,2));
```

(3) Rendering Result [Hollow grum]



Interesting Simulation (Maxima)

6.27.2024
Sohun

1 – 3 3D Implicit Functions

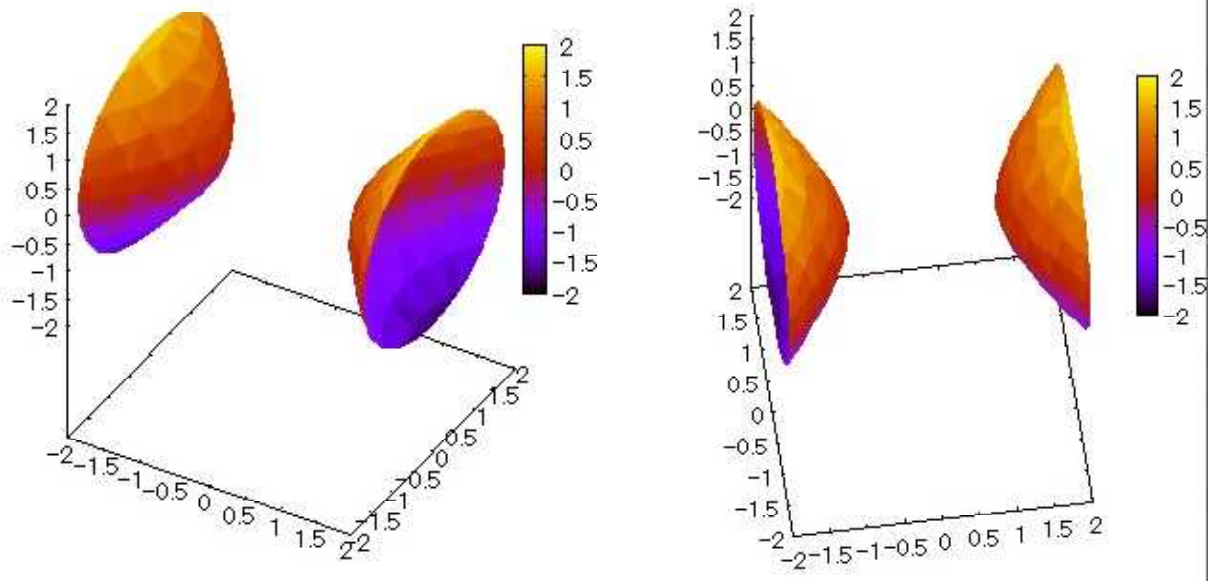
(1) Function Expression

$$x^2 - y^2 - z^2 = 1 \quad (-2 \leq x \leq 2) \quad (-2 \leq y \leq 2) \quad (-2 \leq z \leq 2)$$

(2) Input Expression

```
draw3d(enhanced3d=true,implicit(x^2-y^2-z^2=1,x,-2,2,y,-2,2,z,-2,2));
```

(3) Rendering Result [Two bowls facing each other]



1 – 4 3D Implicit Functions

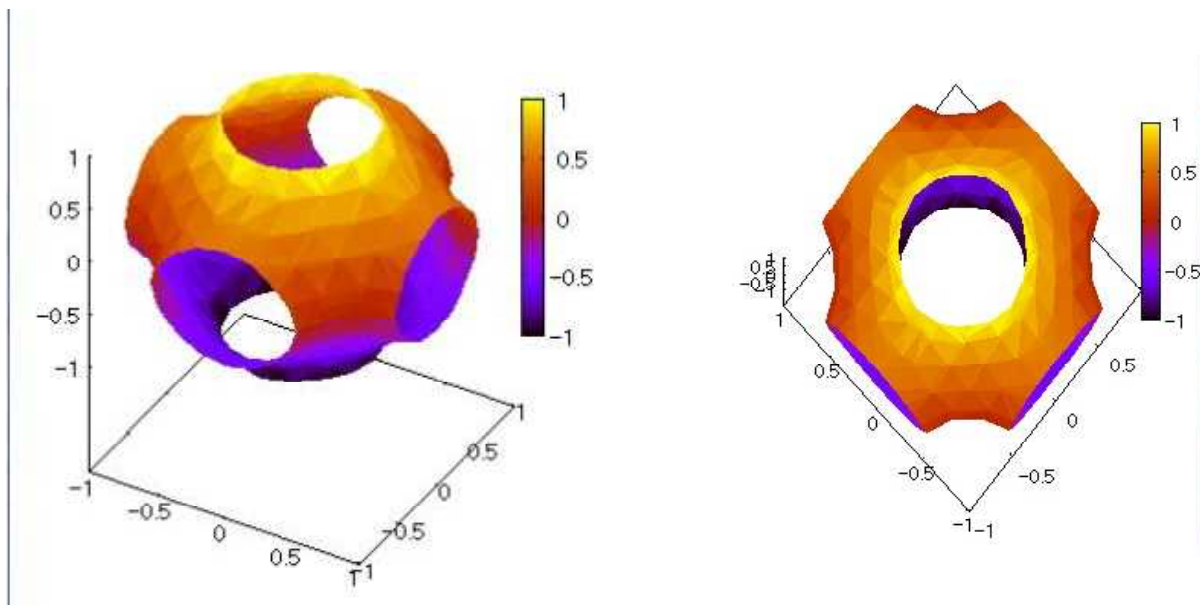
(1) Function Expression

$$(x^2 - 1)^2 + (y^2 - 1)^2 + (z^2 - 1)^2 = 1.5 \quad (-1 \leq x \leq 1) \quad (-1 \leq y \leq 1) \quad (-1 \leq z \leq 1)$$

(2) Input Expression

```
draw3d(enhanced3d=true,implicit((x^2-1)^2+(y^2-1)^2+(z^2-1)^2=1.5,x,-1,1,y,-1,1,z,-1,1));
```

(3) Rendering Result [Sewer pipe joint]



Interesting Simulation (Maxima)

6.27.2024
Sohun

1 – 5 3D Implicit Functions

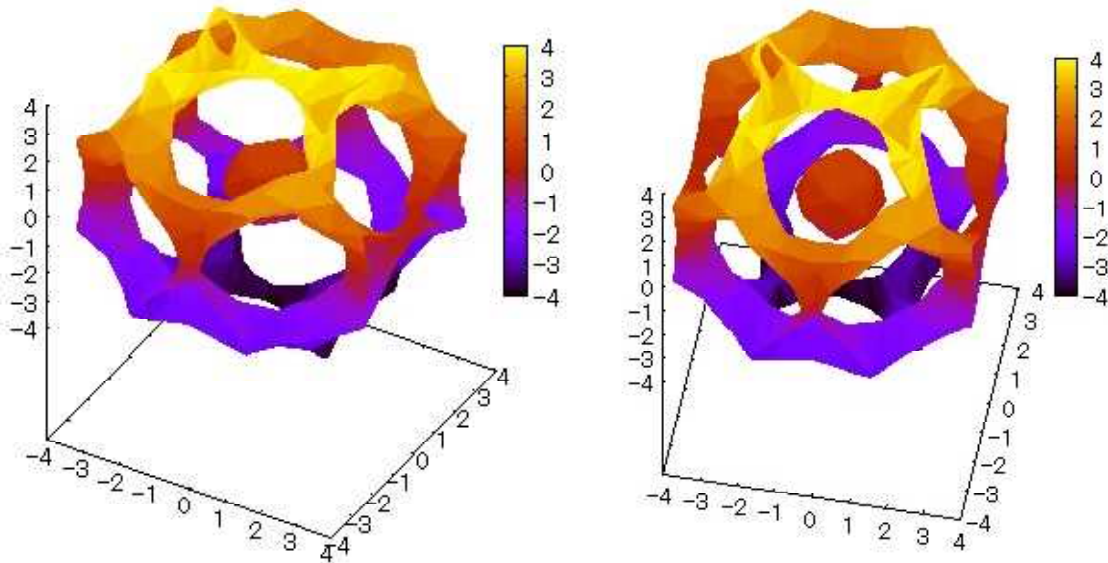
(1) Function Expression

$$\cos(x+\Phi y)+\cos(x-\Phi y)+\cos(y+\Phi z)+\cos(y-\Phi z)+\cos(z+\Phi x)+\cos(z-\Phi x)=2 \quad (\Phi : \text{黄金比})$$

(2) Input Expression

```
draw3d(enhanced3d=true,implicit(cos(x+%phi*y)+cos(x-%phi*y)+cos(y+%phi*z)+cos(y-%phi*z)
+cos(z+%phi*x)+cos(z-%phi*x)=2,x,-4,4,y,-4,4,z,-4,4));
```

(3) Rendering Result [Cells with a central nucleus]



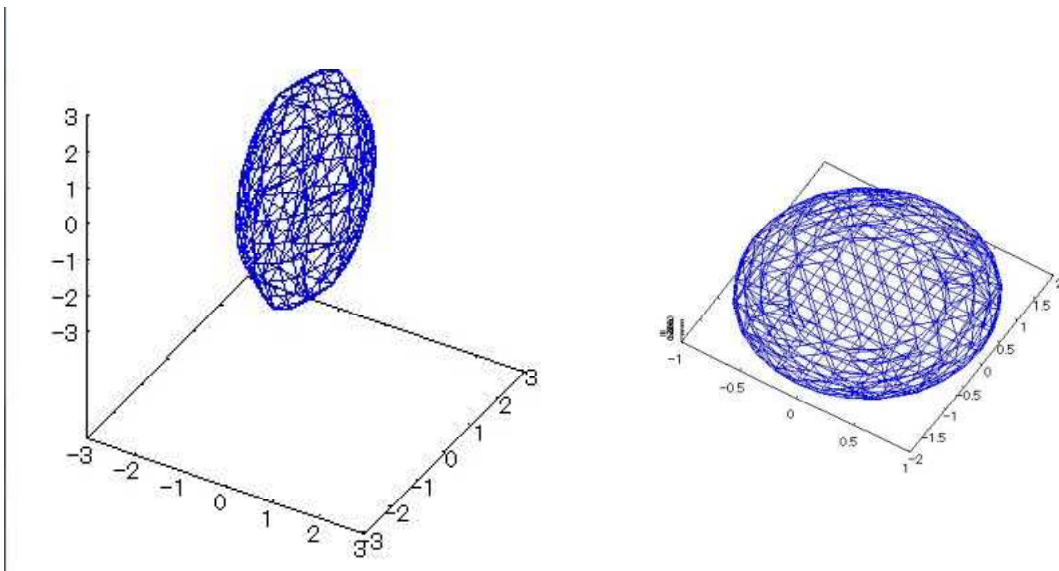
1 – 6 3D Implicit Functions

(1) Function Expression $\frac{x^2}{1^2} + \frac{y^2}{2^2} + \frac{z^2}{3^2} = 1$

(2) Input Expression

```
draw3d(implicit(x^2/1^2+y^2/2^2+z^2/3^2=1,x,-3,3,y,-3,3,z,-3,3));
```

(3) Rendering Result [Ellipsoid]



Interesting Simulation (Maxima)

6.28.2024
Sohun

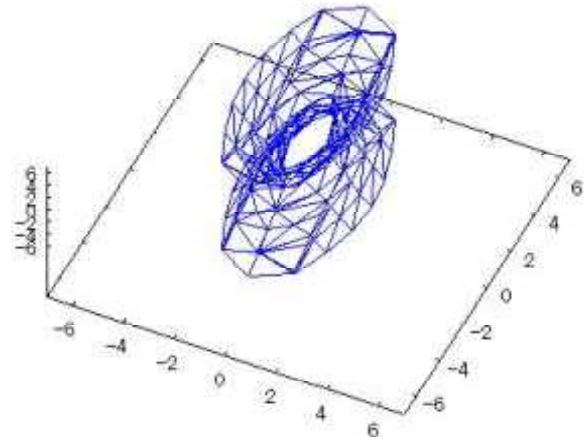
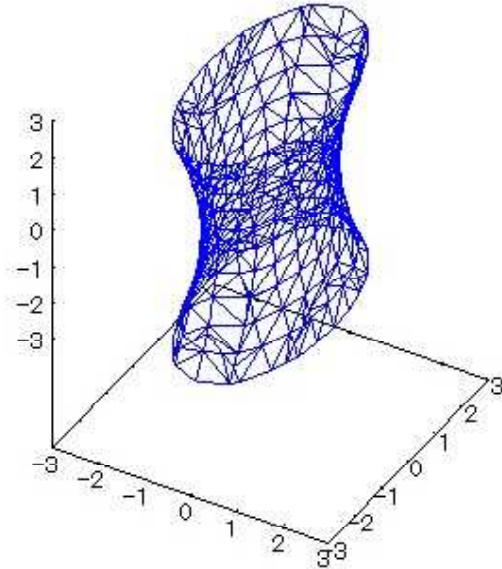
1 – 7 3D Implicit Functions

(1) Function Expression $\frac{x^2}{1^2} + \frac{y^2}{2^2} - \frac{z^2}{3^2} = 1$

(2) Input Expression

```
draw3d(implicit(x^2/1^2+y^2/2^2-z^2/3^2=1,x,-3,3,y,-3,3,z,-3,3));
```

(3) Rendering Result [One-plane hyperbola]



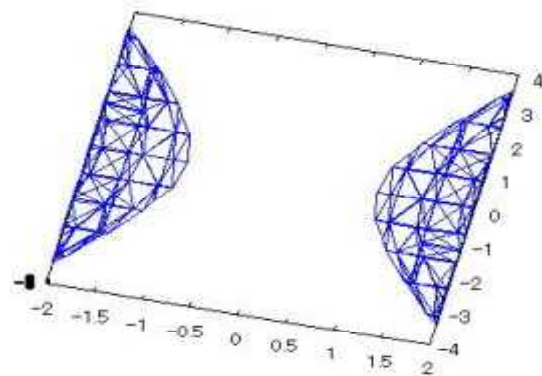
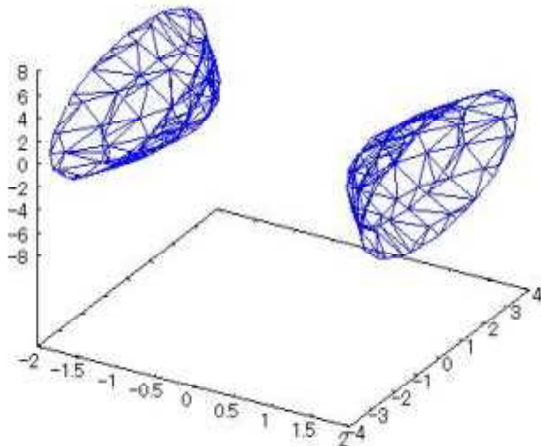
1 – 8 3D Implicit Functions

(1) Function Expression $\frac{x^2}{1^2} - \frac{y^2}{2^2} - \frac{z^2}{3^2} = 1$

(2) Input Expression

```
draw3d(implicit(x^2/1^2-y^2/2^2-z^2/3^2=1,x,-3,3,y,-3,3,z,-3,3));
```

(3) Rendering Result [Two-leaf hyperbola]



Interesting Simulation (Maxima)

6.28.2024
Sohun

2 – 1 3D Explicit Functions

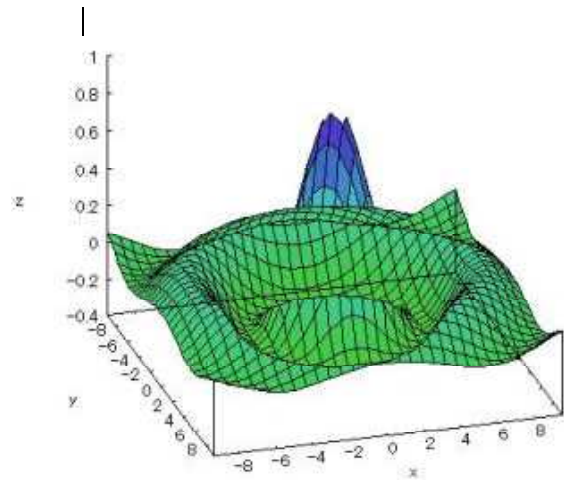
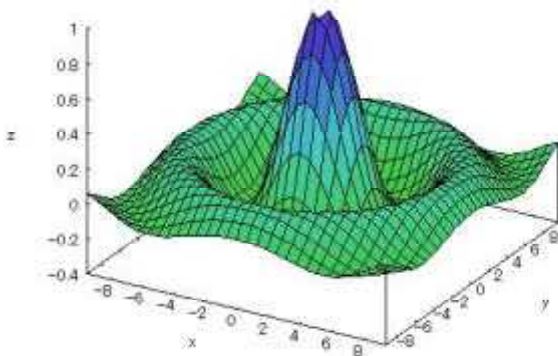
(1) Function Expression

$$z = \frac{\sin \sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2}} \quad (-3\pi \leq x \leq 3\pi) \quad (-3\pi \leq y \leq 3\pi)$$

(2) Input Expression

```
plot3d(sin(sqrt(x^2+y^2))/sqrt(x^2+y^2),[x,-3*%pi,3*%pi],[y,-3*%pi,3*%pi]
,[plot_format,gnuplot],[grid,50,50]);
```

(3) Rendering Result [Mexican Hat]



2 – 2 3D Explicit Functions

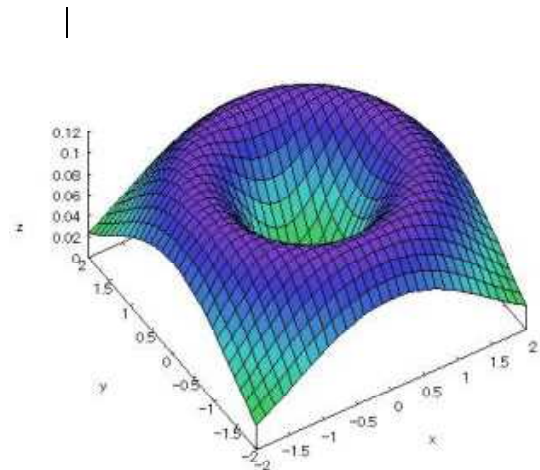
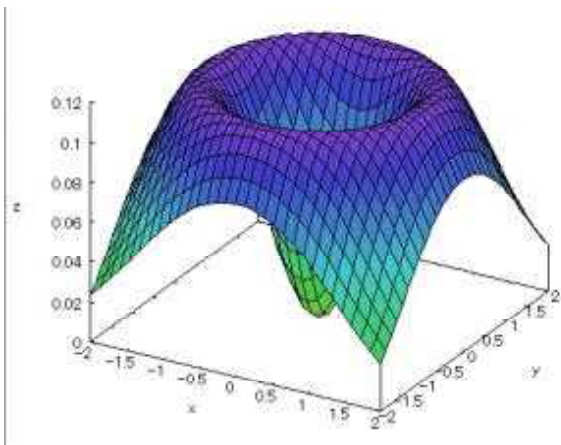
(1) Function Expression

$$z = \frac{x^2 + y^2}{2\pi} e^{-\frac{x^2 + y^2}{2}} \quad (-2 \leq x \leq 2) \quad (-2 \leq y \leq 2)$$

(2) Input Expression

```
plot3d(exp(-(x^2+y^2)/2)*(x^2+y^2)/(2*%pi),[x,-2,2],[y,-2,2]
,[plot_format,gnuplot],[grid,50,50]);
```

(3) Rendering Result [Caldera]



Interesting Simulation (Maxima)

6.29.2024
Sohun

2 – 3 3D Explicit Functions

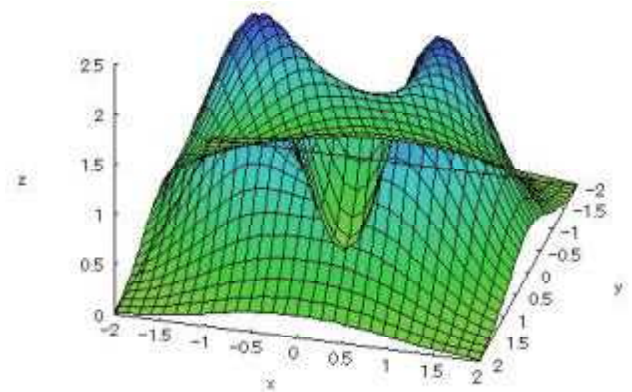
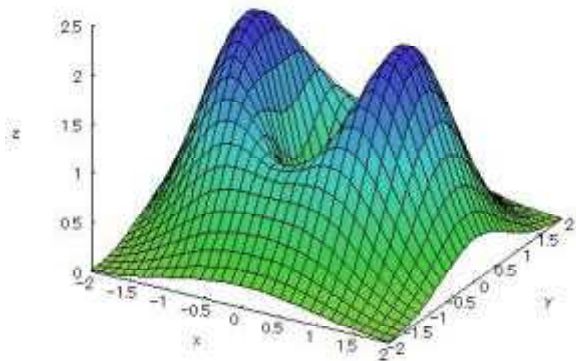
(1) Function Expression

$$z = 3(2x^2 + y^2)e^{-(x^2 + y^2)} \quad (-2 \leq x \leq 2) \quad (-2 \leq y \leq 2)$$

(2) Input Expression

```
plot3d(3*exp(-(x^2+y^2))*(2*x^2+y^2),[x,-2,2],[y,-2,2],
,[plot_format,gnuplot],[grid,50,50]);
```

(3) Rendering Result [Slightly sunken saddle]



2 – 4 3D Explicit Functions

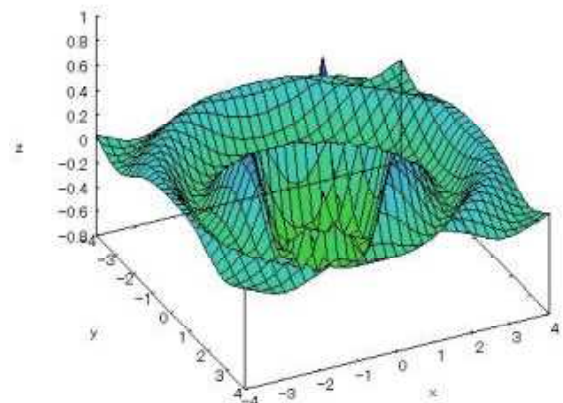
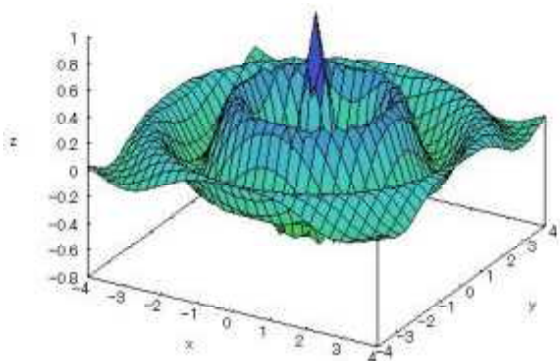
(1) Function Expression

$$e^{\frac{\sqrt{x^2 + y^2}}{2}} \cos(\pi \sqrt{x^2 + y^2}) \quad (-4 \leq x \leq 4) \quad (-4 \leq y \leq 4)$$

(2) Input Expression

```
plot3d(exp(-sqrt(x^2+y^2)/2)*cos(%pi*sqrt(x^2+y^2)),[x,-4,4],[y,-4,4]);
```

(3) Rendering Result [Mexican Hat 2]



Interesting Simulation (Maxima)

6.29.2024
Sohun

2 – 5 D Explicit Functions

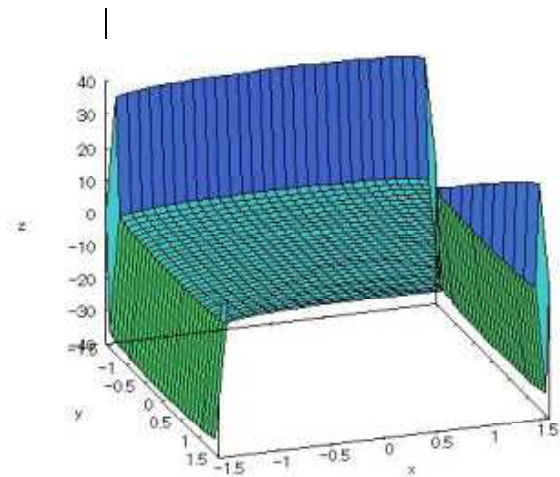
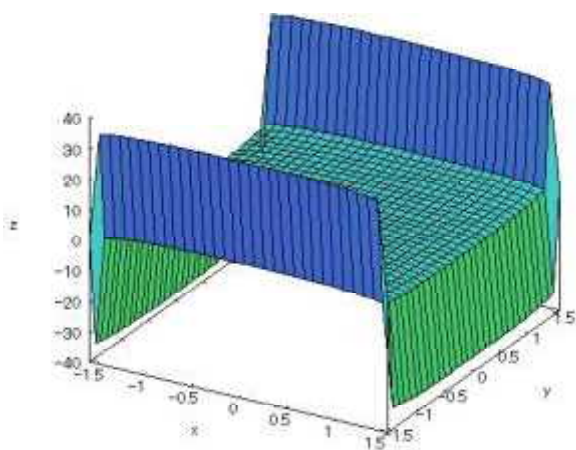
(1) Function Expression

$$z = \log\left(\frac{\cos x}{\cos y}\right) \quad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}, -\frac{\pi}{2} < y < \frac{\pi}{2}\right)$$

(2) Input Expression

```
plot3d(log(cos(x)/cos(y)), [x, -%pi/2, %pi/2], [y, -%pi/2, %pi/2]);
```

(3) Rendering Result [Shark's minimal surfaces]



2 – 6 3D Explicit Functions

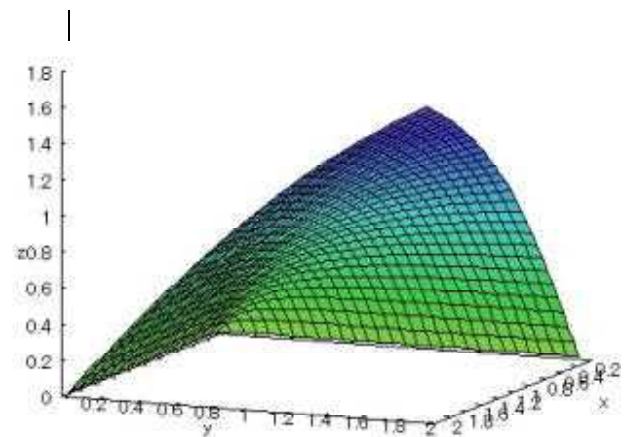
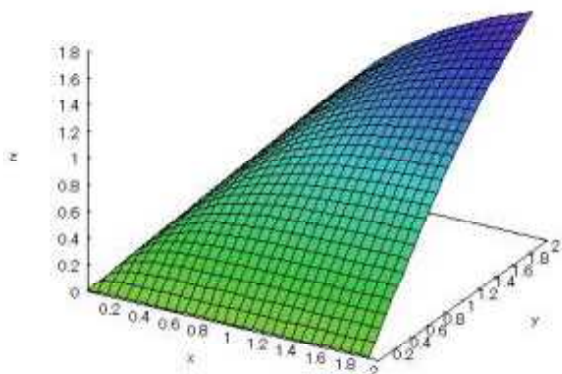
(1) Function Expression

$$z = \frac{1}{2\sqrt{\frac{2}{x^2} + \frac{3}{y^2}}} \quad (0 < x \leq 2) \quad (0 < y \leq 2)$$

(2) Input Expression

```
u(a,x,y,p,q,r):=a*(p*x^(-r)+q*y^(-r))^(1/r);  
plot3d(u(2,x,y,2,3,2), [x, 0.01, 2], [y, 0.01, 2]);
```

(3) Rendering Result [Curving plywood]



Interesting Simulation (Maxima)

7.1.2024
Sohun

2 – 7 3D Explicit Functions

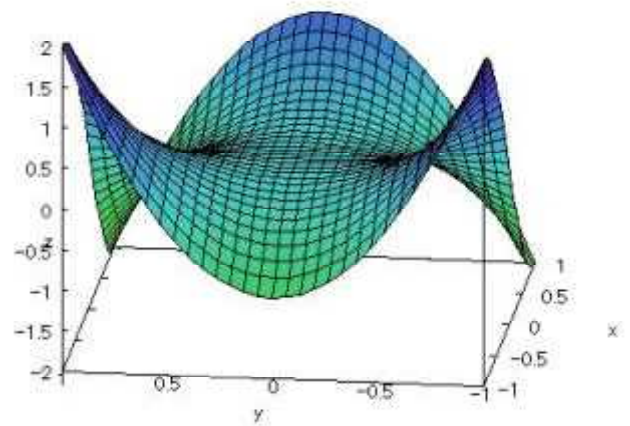
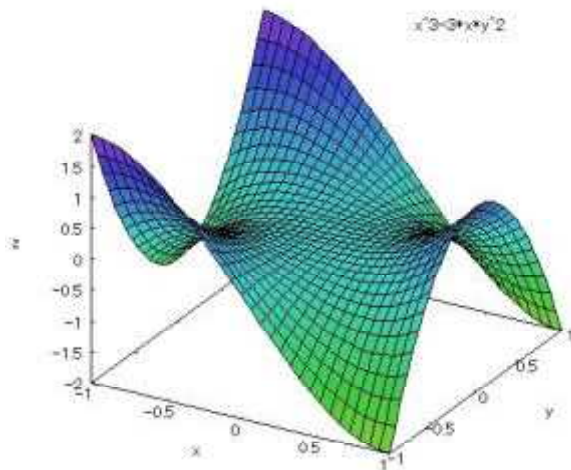
(1) Function Expression

$$z = x^3 - 3xy^2 \quad (-1 \leq x \leq 1, -1 \leq y \leq 1)$$

(2) Input Expression

```
plot3d(x^3-3*x*y^2),[x,-1,1],[y,-1,1];
```

(3) Rendering Result [Flying Carpet]



2 – 8 3D Explicit Functions

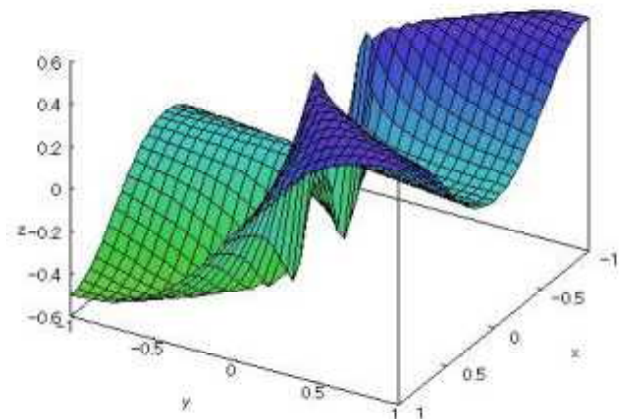
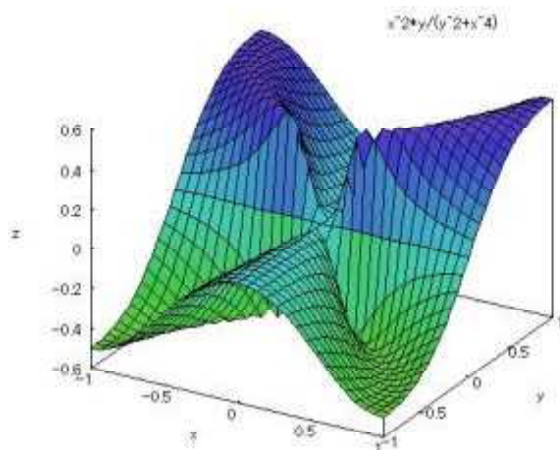
(1) Function Expression

$$z = \frac{x^2 y}{x^4 + y^2}$$

(2) Input Expression

```
plot3d((x^2*y)/(x^4+y^2),[x,-1,1],[y,-1,1]);
```

(3) Rendering Result [Flying Carpet 2]



Interesting Simulation (Maxima)

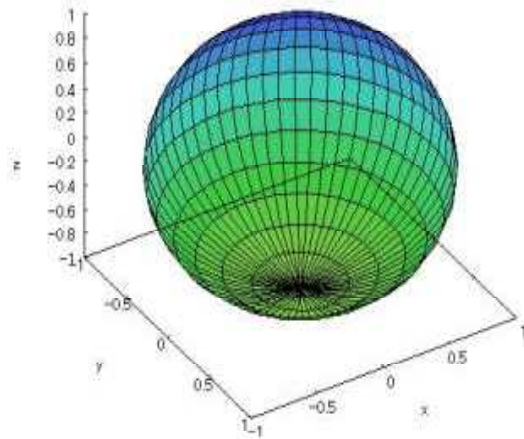
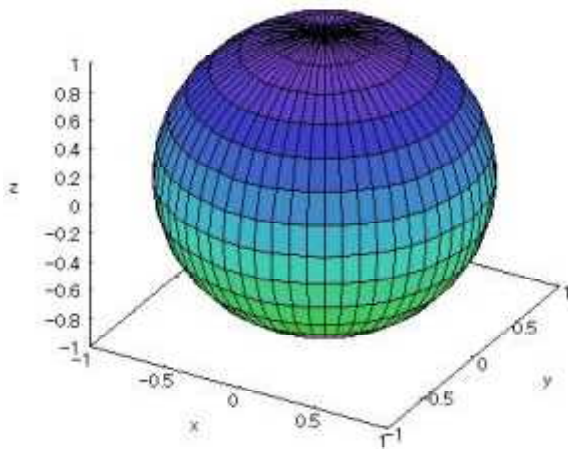
7.1.2024
Sohun

3 – 1 3D Parametric Functions

(1) Function Expression
$$\begin{aligned}x &= \cos s \cdot \cos t \\y &= \cos s \cdot \sin t \\z &= \sin s\end{aligned}\quad (0 \leq s \leq 2\pi, 0 \leq t \leq \pi)$$

(2) Input Expression
`plot3d([cos(s)*cos(t),cos(s)*sin(t),sin(s)],[s,0,2*%pi],[t,0,%pi]);`

(3) Rendering Result [Sphere]

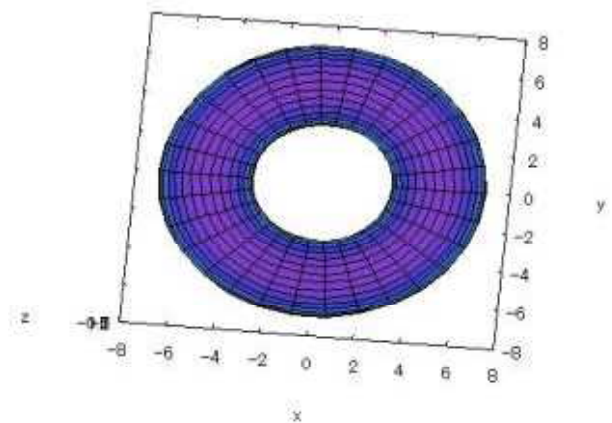
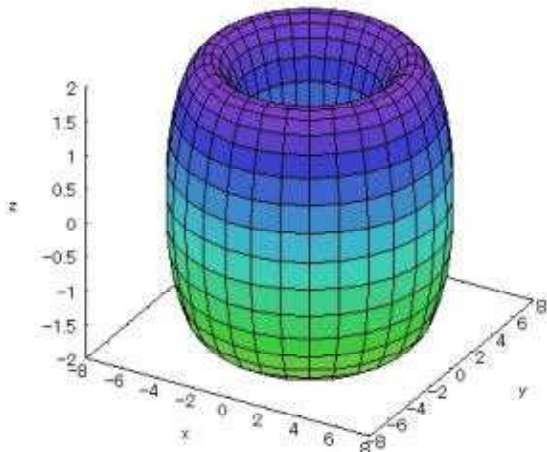


3 – 2 3D Parametric Functions

(1) Function Expression
$$\begin{aligned}x &= (5 + 2 \cos s) \cos t \\y &= (5 + 2 \cos s) \sin t \\z &= 2 \sin s\end{aligned}\quad (0 \leq s \leq 2\pi, 0 \leq t \leq 2\pi)$$

(2) Input Expression
`plot3d([(5+2*cos(s))*cos(t),(5+2*cos(s))*sin(t),2*sin(s)],[s,0,2*%pi],[t,0,2*%pi]);`

(3) Rendering Result [Torus]



Interesting Simulation (Maxima)

7.2.2024
Sohun

3 – 3 3D Parametric Functions

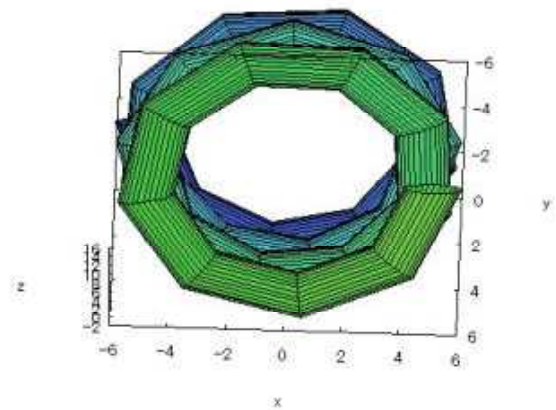
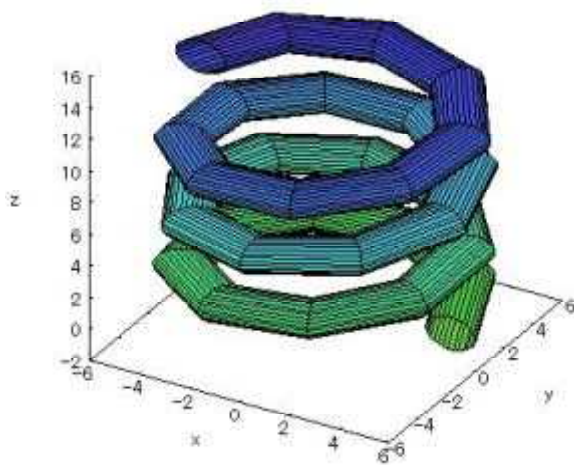
(1) Function Expression

$$\begin{aligned} x &= (5 + \cos s) \cos t \\ y &= (5 + \cos s) \sin t \\ z &= \sin s + 0.6t \quad (0 \leq s \leq 2\pi, 0 \leq t \leq 7\pi) \end{aligned}$$

(2) Input Expression

```
plot3d([(5+cos(s))*cos(t),(5+cos(s))*sin(t),sin(s)+0.6*t],[s,0,2*%pi],[t,0,7*%pi]);
```

(3) Rendering Result [Spring]



3 – 4 3D Parametric Functions

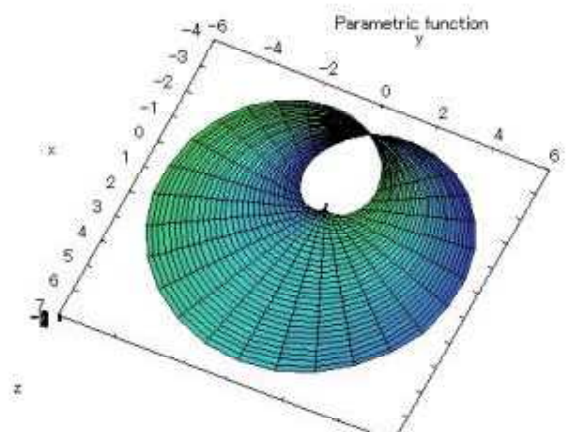
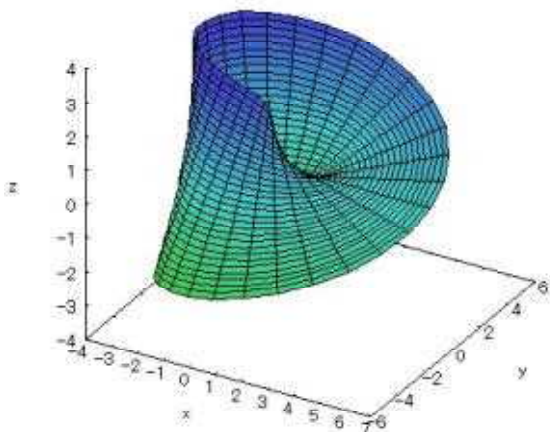
(1) Function Expression

$$\begin{aligned} x &= \cos s (3 + t \cos(s/2)) \\ y &= \sin s (3 + t \cos(s/2)) \\ z &= t \sin(s/2) \quad (-\pi \leq s \leq \pi, -1 \leq t \leq 1) \end{aligned}$$

(2) Input Expression

```
plot3d([cos(s)*(3+t*cos(s/2)),sin(s)*(3+t*cos(s/2)),t*sin(s/2)],[s,-%pi,%pi],[t,-1,1]);
```

(3) Rendering Result [Moebius strip]



Interesting Simulation (Maxima)

7.2.2024
Sohun

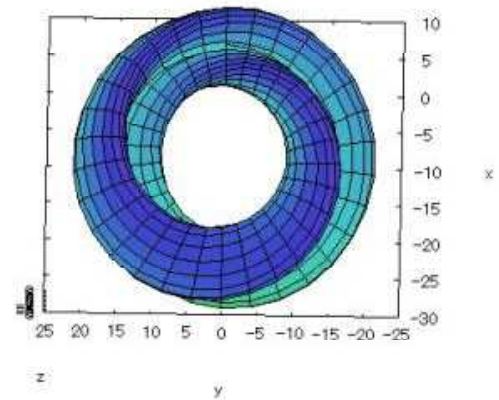
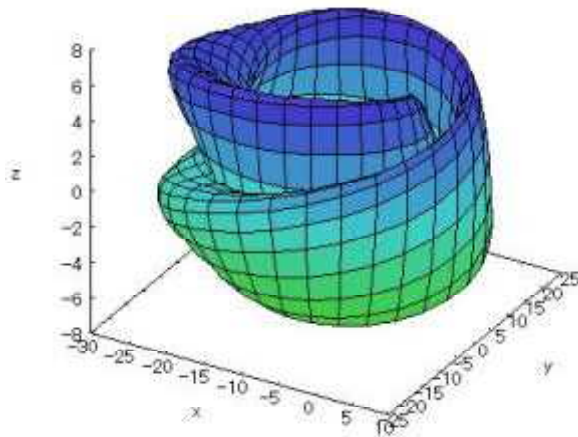
3 – 5 3D Parametric Functions

(1) Function Expression
$$\begin{aligned} x &= 5 \cos s (\cos(s/2) \cos t + \sin(s/2) \sin 2t + 3) - 10 \\ y &= -5 \sin s (\cos(s/2) \cos t + \sin(s/2) \sin 2t + 3) \\ z &= 5 (-\sin(s/2) \cos t + \cos(s/2) \sin 2t) \end{aligned} \quad (-\pi \leq s \leq \pi, -\pi \leq t \leq \pi)$$

(2) Input Expression

```
plot3d([5*cos(s)*(cos(s/2)*cos(t)+sin(s/2)*sin(2*t)+3)-10,
-5*sin(s)*(cos(s/2)*cos(t)+sin(s/2)*sin(2*t)+3.0),
5*(-sin(s/2)*cos(t)+cos(s/2)*sin(2*t))],[s,-%pi,%pi],[t,-%pi,%pi]);
```

(3) Rendering Result [Klein's Bottle]



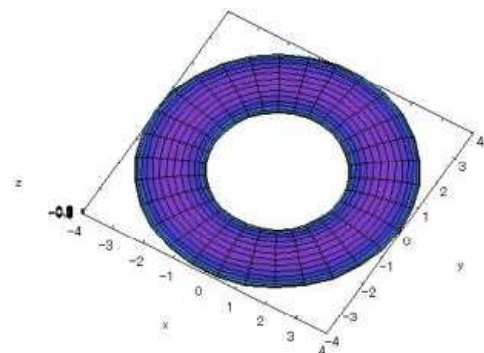
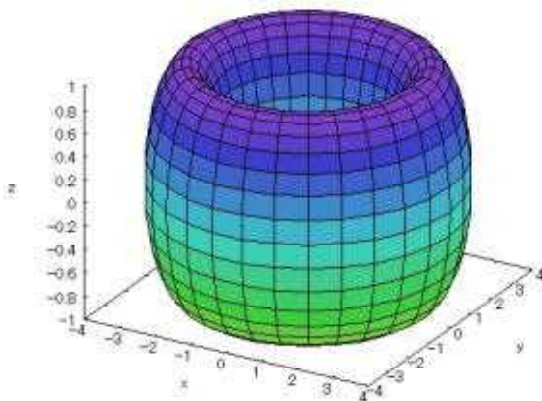
3 – 6 3D Parametric Functions

(1) Function Expression
$$\begin{aligned} x &= \cos s (3 + \cos t) \\ y &= \sin s (3 + \cos t) \\ z &= \sin t \end{aligned} \quad (0 \leq s \leq 2\pi, 0 \leq t \leq 2\pi)$$

(2) Input Expression

```
plot3d([cos(s)*(3+cos(t)),sin(s)*(3+cos(t)),sin(t)],[s,0,2*%pi],[t,0,2*%pi]);
```

(3) Rendering Result [Torus]



Interesting Simulation (Maxima)

7.2.2024
Sohun

3 – 7 3D Parametric Functions

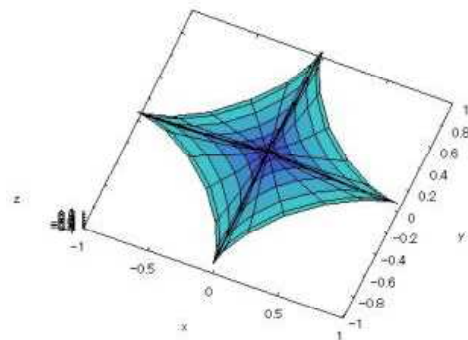
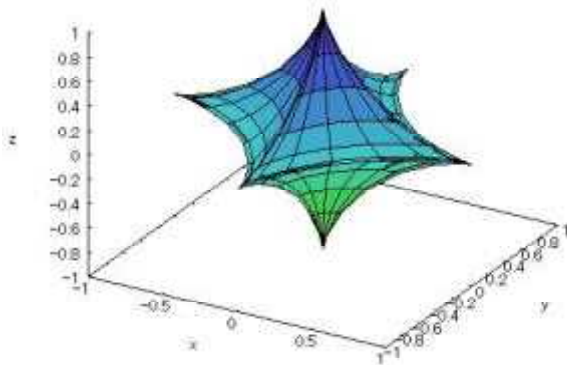
(1) Function Expression

$$\begin{aligned} x &= \cos^3 s \cos^3 t \\ y &= \sin^3 s \cos^3 t \\ z &= \sin^3 t \end{aligned} \quad (0 \leq s \leq 2\pi, 0 \leq t \leq 2\pi)$$

(2) Input Expression

```
plot3d([cos(s)^3*cos(t)^3,sin(s)^3*cos(t)^3,sin(t)^3],[s,0,2*%pi],[t,0,2*%pi]);
```

(3) Rendering Result [Asteroid sphere]



3 – 8 3D Parametric Functions

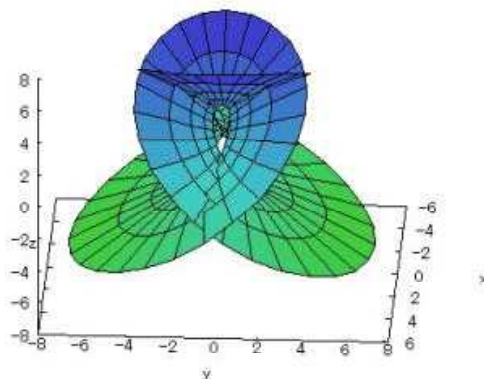
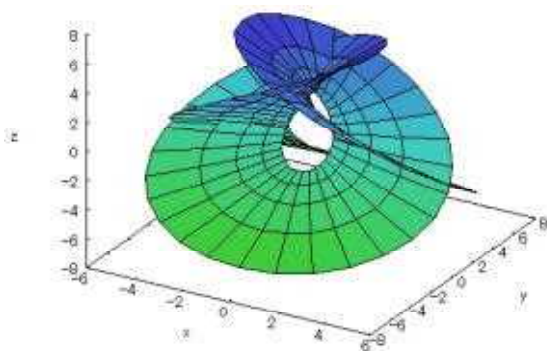
(1) Function Expression

$$\begin{aligned} x &= 2 \sinh s \cos t - (2/3) \sinh^3 s \cos^3 t \\ y &= 2 \sinh s \sin t - (2/3) \sinh^3 s \sin^3 t \\ z &= 2 \cosh^2 s \cos^2 t \end{aligned} \quad (0.3 \leq s \leq 0.9, 0 \leq t \leq 2\pi)$$

(2) Input Expression

```
plot3d([2*sinh(s)*cos(t)-(2/3)*sinh(3*s)*cos(3*t),
2*sinh(s)*sin(t)-(2/3)*sinh(3*s)*sin(3*t),
2*cosh(2*s)*cos(2*t)],[s,0.3,0.9],[t,0,2*%pi],[grid,4,72]);
```

(3) Rendering Result [Henneberg's minimal surfaces]



Interesting Simulation (Maxima)

7.2.2024
Sohun

3 – 9 3D Parametric Functions

(1) Function Expression

$$x = 3 \cos u + 5 \cos 3u + \frac{3(\cos u + 5 \cos 3u) \cos v}{2\sqrt{234 + 90 \cos 2u}} - \frac{3 \cos 6u (\sin u + 5 \sin 3u) \sin v}{2\sqrt{13 + 5 \cos 2u} \sqrt{22 + 5 \cos 2u + 9 \cos 12u}}$$

$$y = 3 \sin u + 5 \sin 3u + \frac{3 \cos v (\sin u + 5 \sin 3u)}{2\sqrt{234 + 90 \cos 2u}} + \frac{3(5 \cos 3u + \cos 5u + \cos 7u + 5 \cos 9u) \sin v}{4\sqrt{13 + 5 \cos 2u} \sqrt{22 + 5 \cos 2u + 9 \cos 12u}}$$

$$z = 3 \sin 6u - \frac{\sqrt{13 + 5 \cos 2u} \sin v}{2\sqrt{22 + 5 \cos 2u + 9 \cos 12u}}$$

$$(0 \leq u \leq 2\pi, 0 \leq v \leq 2\pi)$$

(2) Input Expression

```
plot3d([3*cos(u) + 5*cos(3*u) + (3*(cos(u) + 5*cos(3*u)) * cos(v)) / (2*sqrt(234 + 90*cos(2*u))) - (3*cos(6*u) * (sin(u) + 5*sin(3*u)) * sin(v)) / (2*sqrt(13 + 5*cos(2*u)) * sqrt(22 + 5*cos(2*u) + 9*cos(12*u))), 3*sin(u) + 5*sin(3*u) + (3*cos(v) * (sin(u) + 5*sin(3*u))) / (2*sqrt(234 + 90*cos(2*u))) + (3*(5*cos(3*u) + cos(5*u) + cos(7*u) + 5*cos(9*u)) * sin(v)) / (4*sqrt(13 + 5*cos(2*u)) * sqrt(22 + 5*cos(2*u) + 9*cos(12*u))), 3*sin(6*u) - (sqrt(13 + 5*cos(2*u)) * sin(v)) / (2*sqrt(22 + 5*cos(2*u) + 9*cos(12*u)))] , [u, 0, 2*%pi] , [v, 0, 2*%pi] , [grid, 80, 8]);
```

(3) Rendering Result [String]

