

Modeling and Simulation III

3.6.2024
Sohun

1 Playing Cards that Are Multiples of 3

(1) Experiment overview

Here is a deck of cards.

The deck uses 52 playing cards ,
excluding the two jokers.

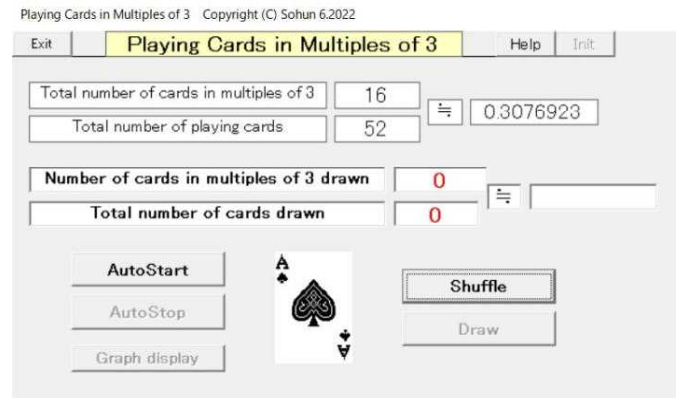
There are four of each of the multiples
of 3 ; spades , clovers , diamonds ,
and hearts making a total of 16 cards.

When you draw a card from this deck of
52 cards , consider the probability that that
card is a multiple of 3.

Since 16 of the 52 cards are multiples of
3 , mathematically the probability is $16/52$.

However , in reality , if you draw and
put back one at a time , 52 times , it is
not guaranteed that you will exactly draw
16 cards that are multiples of 3.

So how does this relate to mathematically calculated theoretical probability of $16/52$?



(2) Experimental result (VB version simulation)

【Experiment day】

March 6 . 2024

【PC used】

Lavie NX850/N

【Software used】

Self-made software

『toramp 6』

【Method of operation】

To do this manually , click the [Shuffle] button , then click the [Draw] button.

To do this automatically , click the [AutoStart] button , then click the [AutoStop] button.

Clicking the [Graph display] button will display a graph showing the relationship between
the total number of cards drawn and the percentage of cards that were multiples of 3 .

Click the [Init] button to restart the experiment from the beginning.

【Consideration】

In the 1st experiment , I draw and put back one card at a time 1000 times.

I drew cards that were multiples of three 289 times.

The ratio of drawing a card that is a multiple of 3 is $289 \div 1000 = 0.289$.

The mathematically calculated theoretical probability is $16 \div 52 = 0.3076923$.

The ratio of drawing a card that is a multiple of 3 , 0.289 is close to the mathematically
calculated theoretical probability of 0.3076923.

In the 2nd experiment , I draw and put back one card at a time 1000 times , too.

I drew cards that were multiples of three 304 times.

The ratio of drawing a card that is a multiple of 3 is $304 \div 1000 = 0.304$.

The ratio of drawing a card that is a multiple of 3 , 0.304 is close to the mathematically
calculated theoretical probability of 0.3076923.

The graphs for experiments ① and ② have the number of cards drawn on the horizontal
axis and the percentage of cards drawn that were multiples of 3 at that time on the vertical axis.

The more cards you draw , the closer your chances of drawing a card that is a multiple of
3 become to the mathematically calculated theoretical probability of $16/52$ (0.3076923) .

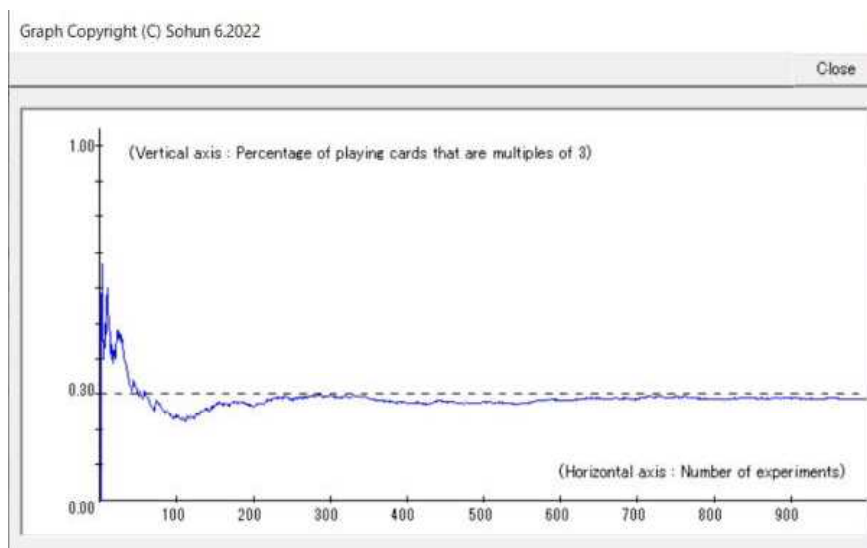
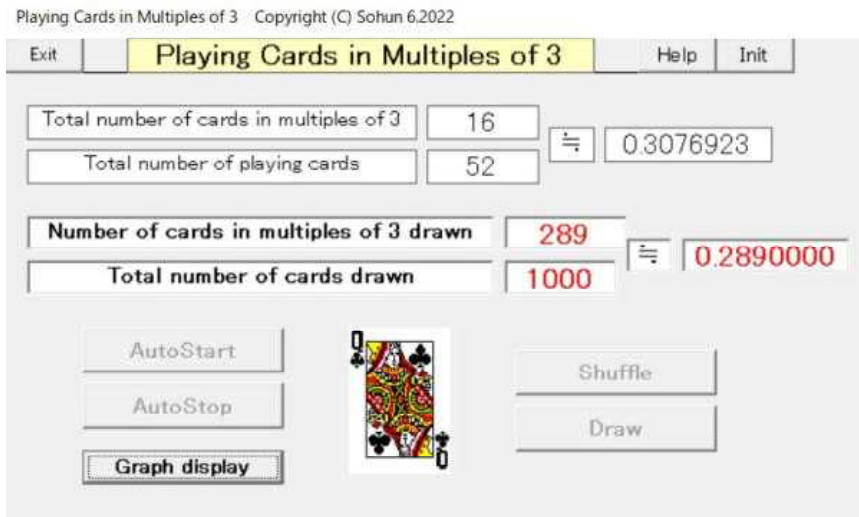
Modeling and Simulation III

3.6.2024
Sohun

1 Playing Cards that Are Multiples of 3

(2) Experimental result (VB version simulation)

① 1st experiment



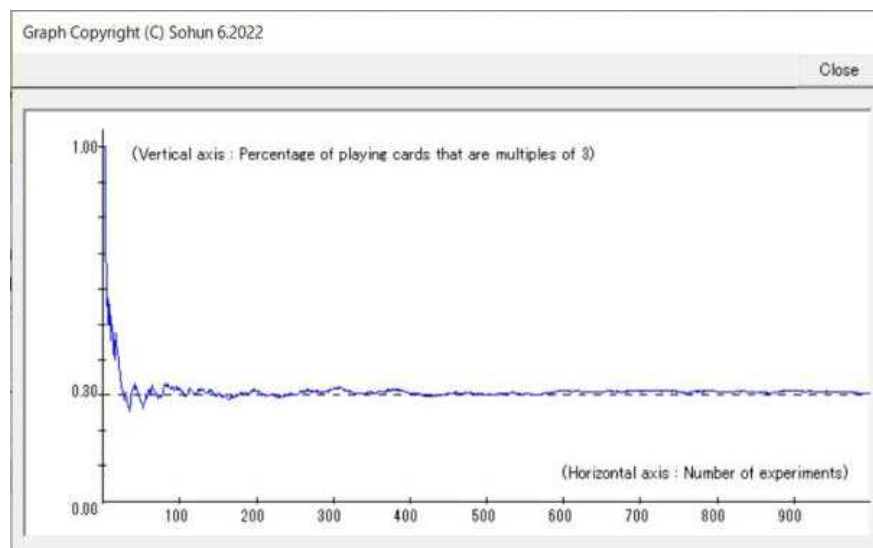
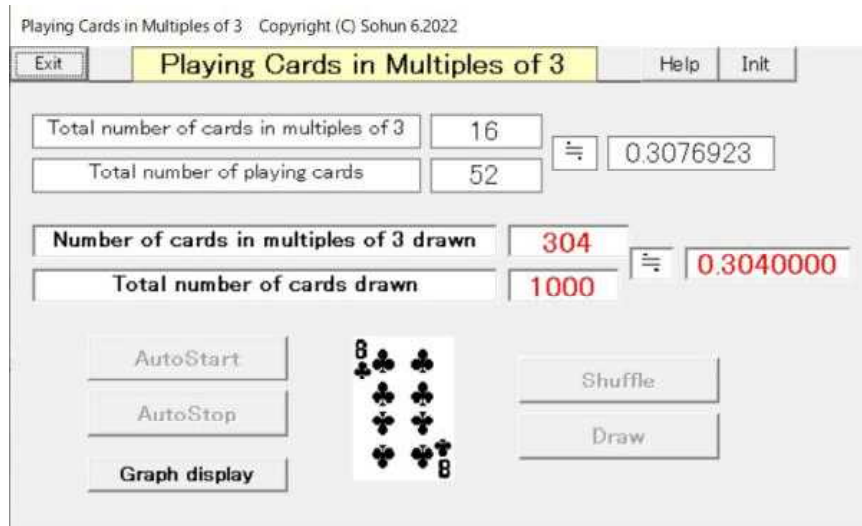
Modeling and Simulation III

3.6.2024
Sohun

1 Playing Cards that Are Multiples of 3

(2) Experimental result (VB version simulation)

② 2nd experiment



Modeling and Simulation III

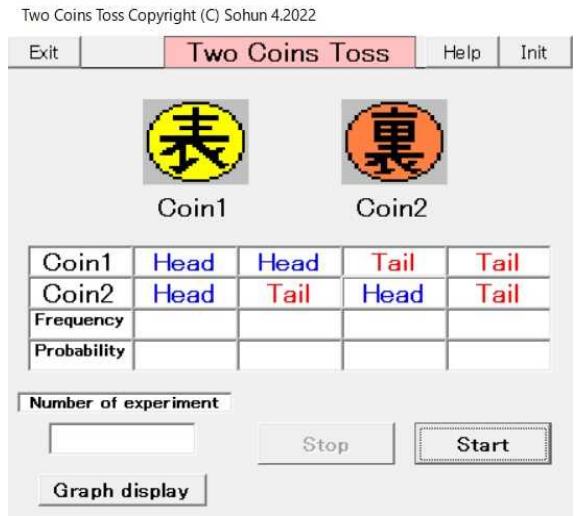
3.8.2024
Sohun

2 Two Coins Toss

(1) Experiment overview

Let's toss two coins at the same time and think about how they will come up heads and tails.

Let these two coins be coin1 and coin2, respectively. There are four ways to come out: heads and heads, heads and tails, tails and heads, and tails and tails. Mathematically, the probability of each of these four coins occurring is $1/4$. However, in reality, when two coins are tossed four times at the same times, heads and heads, heads and tails, tails and heads, and tails and tails don't necessarily occur once each. So what is the relationship with the theoretical probability of $1/4$ calculated mathematically?



(2) Experimental result (VB version simulation)

【Experiment day】

March 8 . 2024

【PC used】

Lavie NX850/N

【Software used】

Self-made software

『two coins toss 6』

【Method of operation】

[Start] Click the button to start the experiment.

[Stop] Click the button to stop the experiment.

[Graph display] Clicking the button will display a graph showing the relationship between the number of experiments and the proportion of heads and heads, heads and tails, tails and heads, and tails and tails.

[Init] Click the button to restart the experiment from the beginning.

【Consideration】

In experiment ①, coins1 and 2 were tossed simultaneously 3965 times.

Both coin1 and coin2 came up heads 997 times, coin1 came up heads and coin2 came up tails 996 times, coin1 came up tails and coin2 came up heads 972 times, both coin1 and coin2 came up tails 1000 times. Also the percentage of coin1 and coin2 coming up heads is 0.251, the percentage of coin1 coming up heads and coin2 coming up tails is 0.251, the percentage of coin1 coming up tails and coin2 coming up heads is 0.245, and the percentage of coin1 and coin2 coming up tails is 0.252.

In experiment ②, coins 1 and 2 were tossed simultaneously 3892 times.

Both coin1 and coin2 came up heads 946 times, coin1 came up heads and coin2 came up tails 1000 times, coin1 came up tails and coin2 came up heads 953 times, both coin1 and coin2 came up tails 993 times. Also the percentage of coin1 and coin2 coming up heads is 0.243, the percentage of coin1 coming up heads and coin2 coming up tails is 0.257, the percentage of coin1 coming up tails and coin2 coming up heads is 0.245, and the percentage of coin1 and coin2 coming up tails is 0.255.

From the graph, we can see that when the coin1 and coin2 are tossed at the same time, the ratio of heads to heads, heads to tails, tails to heads, and tails to tails approaches $1/4$ (0.25) as the number of tosses increases.

Modeling and Simulation III

3.8.2024
Sohun


2 Two Coins Toss

(2) Experimental result (VB version simulation)

Experiment ①

Two Coins Toss Copyright (C) Sohun 4.2022

Exit Two Coins Toss Help Init



Coin1 Coin2

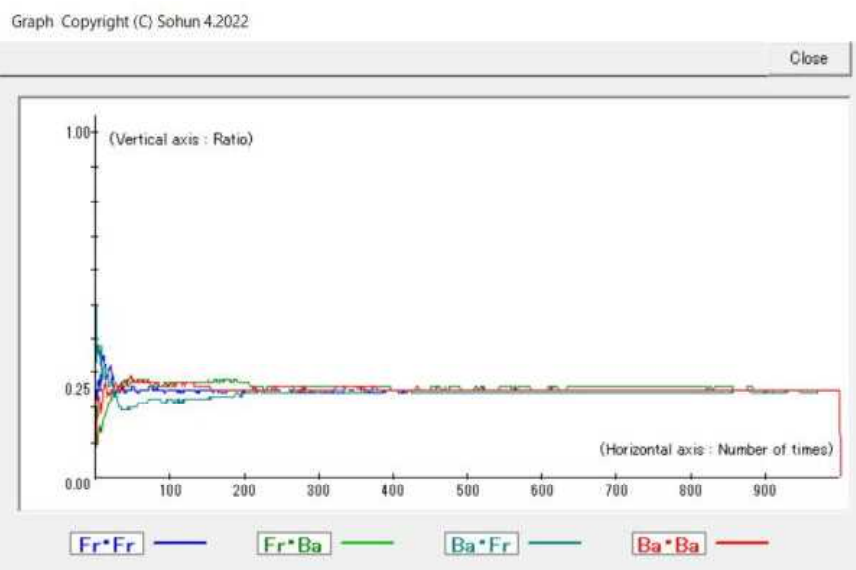
| | | | | |
|-------------|-------|-------|-------|-------|
| Coin1 | Head | Head | Tail | Tail |
| Coin2 | Head | Tail | Head | Tail |
| Frequency | 997 | 996 | 972 | 1000 |
| Probability | 0.251 | 0.251 | 0.245 | 0.252 |

Number of experiment

3965

Stop Start

Graph display



Modeling and Simulation III

3.8.2024
Sohun


2 Two Coins Toss

(2) Experimental result (VB version simulation)

Experiment ②

Two Coins Toss Copyright (C) Sohun 4.2022

Exit Two Coins Toss Help Init



Coin1 Coin2

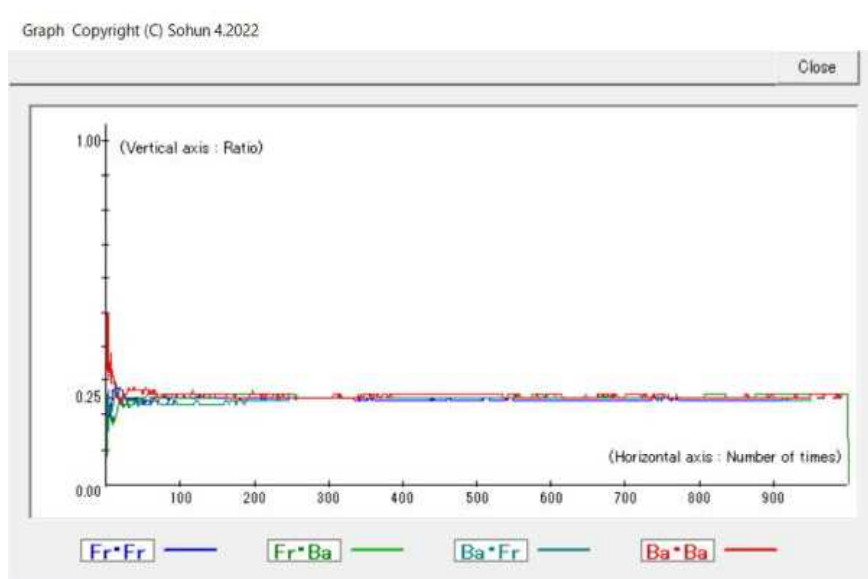
| | | | | |
|-------------|-------|-------|-------|-------|
| Coin1 | Head | Head | Tail | Tail |
| Coin2 | Head | Tail | Head | Tail |
| Frequency | 946 | 1000 | 953 | 993 |
| Probability | 0.243 | 0.257 | 0.245 | 0.255 |

Number of experiment

3892

Stop Start

Graph display



Modeling and Simulation III

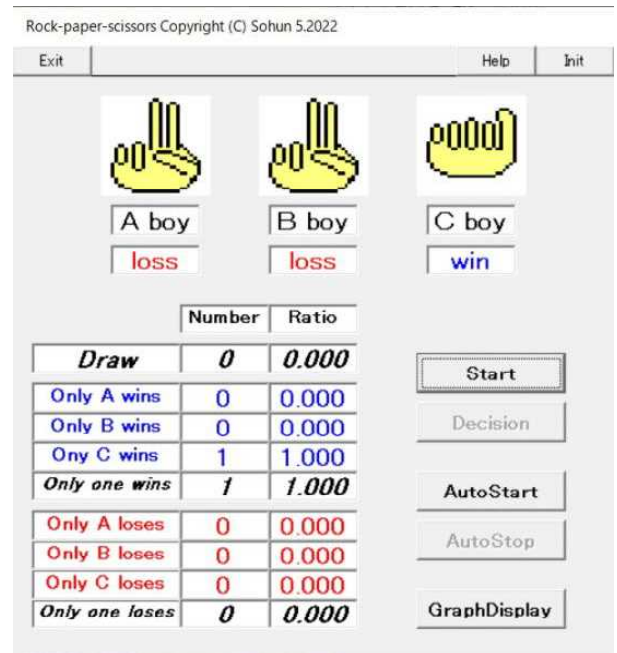
3.10.2024
Sohun

3 Three People Playing Rock-Paper-Scissors

(1) Experiment overview

Three people, A, B, C play Rock-Paper-Scissors once. Draw has 9 ways: for A, B, and C: Rock/Rock/Rock, Paper/Paper/Paper, Scissors/Scissors/Scissors, Rock/Scissors/Paper, Rock/Paper/Scissors, Paper/Scissors/Rock, Paper/Rock/Scissors, Scissors/Rock/Paper, Scissors/Paper/Rock. If A wins alone, there are three ways for A, B, and C: Rock/Scissors/Scissor, Scissor/Paper/Paper, Paper/Rock/Rock. Similarly for B, C, there are three ways in which only one person can win. If A loses alone, there are 3 ways for A, B, and C: Rock/Paper/Paper, Scissors/Rock/Rock, Paper/Scissors/Scissors. Similarly for B, C, there are three ways in which only one person can lose.

In other words, when three people play Rock-Paper-Scissors once, there are a total of 27 ways to play Rock-Paper-Scissors. For example, here are 9 ways to become a draw so the mathematical probability is $9/27$, which can be reduced to $1/3$ (0.333). However, in reality, if you play Rock-Paper-Scissors three times, there will not be only one draw. So, what is the relationship with the mathematically calculated theoretical probability of a draw of $1/3$?



(2) Experimental result (VB version simulation)

【Experiment day】

March 10, 2024

【PC used】

Lavie NX850/N

【Software used】

Self-made software

『jyanken 6』

【Method of operation】

If you want to do it manually, click the [Start] button, then click the [Decision] button.

If you want to do it automatically, click the [AutoStart] button, then click the [AutoStop] button.

Click the [GraphDisplay] button to display graphs of the percentage for cases where there is a draw, only one person wins, and only one person loses.

Click the [Init] button to restart the experiment from the beginning.

【Consideration】

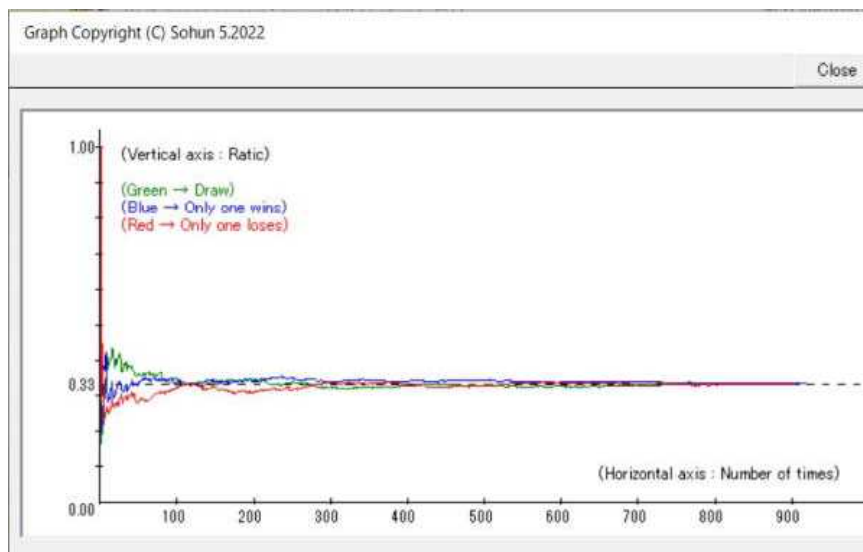
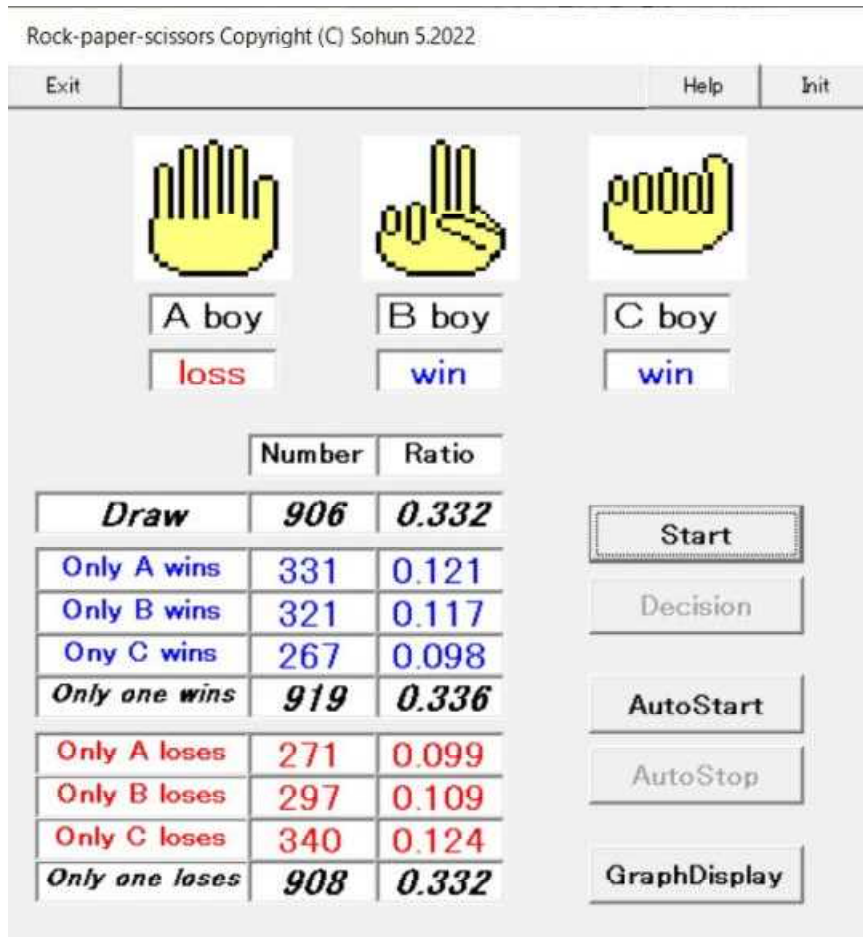
In the experiment, three people A, B, and C played Rock-Paper-Scissors 2733 times. There were 906 draws, and the percentage was 0.332. Only one person won 919 times, and the percentage was 0.336. Only one person lost 908 times and the percentage was 0.332. The mathematical probability of a draw is $9/27$ (0.333). The mathematical probability that only one person wins is $9/27$ (0.333). The mathematical probability that only one person loses is $9/27$ (0.333). From the graph of the experimental results, we can see that when three people play Rock-Paper-Scissors a lot, the percentage of draws, the percentage of only one player winning, and only one player losing respectively approach 0.33.

Modeling and Simulation III

3.10.2024
Sohun

3 Three People Playing Rock-Paper-Scissors

(2) Experimental result (VB version simulation)



Modeling and Simulation III

3.12.2024
Sohun

4 Two Dice with an Odd Product

(1) Experiment overview

Think about what will happen if you throw two large and small dice at the same time.

Let's call two dice large and small, respectively. There are thirty six ways to get the numbers : $1 \cdot 1$, $1 \cdot 2$, $1 \cdot 3$, $1 \cdot 4$, $1 \cdot 5$, $1 \cdot 6$, $2 \cdot 1$, $2 \cdot 2$, $2 \cdot 3$, $2 \cdot 4$, $2 \cdot 5$, $2 \cdot 6$, $3 \cdot 1$, $3 \cdot 2$, $3 \cdot 3$, $3 \cdot 4$, $3 \cdot 5$, $3 \cdot 6$, $4 \cdot 1$, $4 \cdot 2$, $4 \cdot 3$, $4 \cdot 4$, $4 \cdot 5$, $4 \cdot 6$, $5 \cdot 1$, $5 \cdot 2$, $5 \cdot 3$, $5 \cdot 4$, $5 \cdot 5$, $5 \cdot 6$, $6 \cdot 1$, $6 \cdot 2$, $6 \cdot 3$, $6 \cdot 4$, $6 \cdot 5$, and $6 \cdot 6$.

Also, there are nine ways that the product of two large and small dice can be an odd number : $1 \cdot 1$, $1 \cdot 3$, $1 \cdot 5$, $3 \cdot 1$, $3 \cdot 3$, $3 \cdot 5$, $5 \cdot 1$, $5 \cdot 3$, and $5 \cdot 5$.

Therefore, mathematically, the probability that the product of two large and small dice will be an odd number is $9/36$, which is reduced to $1/4$ (0.25).

However, in reality, if you toss two coins at the same time four times, it isn't necessarily mean that the product of the two dice will be an odd number only once. So, what is the relationship with the mathematically calculated theoretical probability of $1/4$?

| Large \ Small | 1 | 2 | 3 | 4 | 5 | 6 |
|---------------|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 |

(2) Experimental result (VB version simulation)

【Experiment day】

March 12 . 2024

【PC used】

Lavie NX850/N

【Software used】

Self-made software

『two dice 6』

【Method of operation】

If you want to experiment manually, click the [Throw the dice] button, then click the [Stop the dice] button.

If you want to experiment automatically, click the [Automatic] button, then click the [Stop] button.

Click the [Graph display] button to display a graph showing the relationship between the number of experiments and the ratio of the product of the two dice to an odd number.

Click the [Init] button to restart the experiment from the beginning.

【Consideration】

In the experiment, two large and small dice were thrown simultaneously 1000 times. When the product is an odd number, in order of large dice and small dice, $1 \cdot 1$ are 32 times, $1 \cdot 3$ are 32 times, $1 \cdot 5$ are 20 times, $3 \cdot 1$ are 22 times, $3 \cdot 3$ are 28 times, $3 \cdot 5$ are 35 times, $5 \cdot 1$ are 19 times, $5 \cdot 3$ are 20 times, $5 \cdot 5$ are 48 times for a total of 256 times. The ratio of the product of the two dice being an odd number was 0.256.

The mathematical probability that the product of two dice will be an odd number is $9/36$ (0.25). From the graph of the experimental results, we can see that if you throw two large and small dice at the same time many times, the ratio of the product of the two large and small dice to an odd number approaches 0.25.

Modeling and Simulation III



3.12.2024
Sohun

4 Two Dice with an Odd Product

(2) Experimental result (VB version simulation)

Two Dice with Odd Product Copyright (C) Sohun 6.2022

Exit Two Dice with Odd Product Help Init

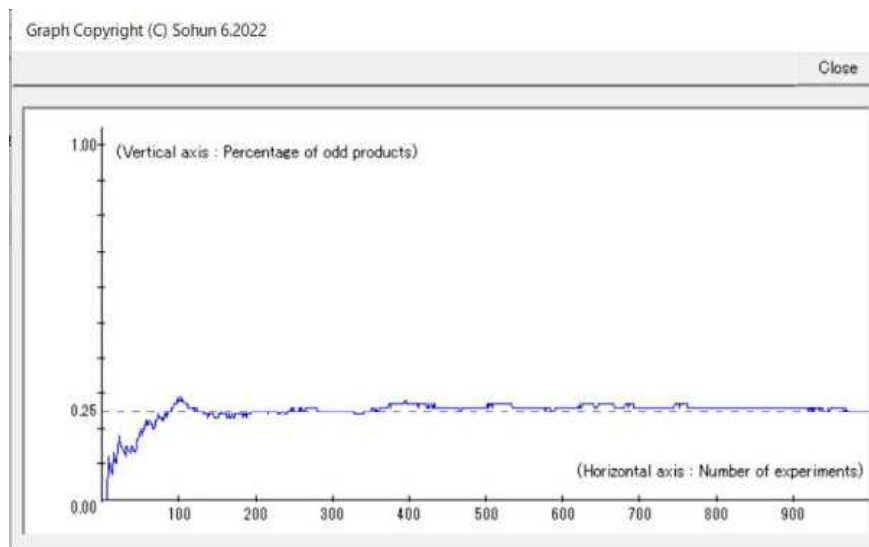
Large dice Small dice

| Large \ Small | 1 | 2 | 3 | 4 | 5 | 6 |
|---------------|----|----|----|----|----|----|
| 1 | 32 | 29 | 32 | 28 | 20 | 27 |
| 2 | 22 | 21 | 37 | 34 | 33 | 26 |
| 3 | 22 | 25 | 28 | 35 | 35 | 37 |
| 4 | 33 | 24 | 14 | 34 | 28 | 28 |
| 5 | 19 | 25 | 20 | 22 | 48 | 21 |
| 6 | 27 | 25 | 26 | 29 | 28 | 26 |

Number of experiments: 1000
Number of odd products: 256
Percentage of odd products: 0.256

Throw the dice
Stop the dice

Graph display Automatic Stop



Modeling and Simulation III

3.13.2024
Sohun

5 Collatz Problem

(1) Experiment overview

Any natural number will do, so if the number is even, divide it by 2, if it is odd, multiply it by 3 and add 1, and repeat.

So is it true that no matter what number we start with, it always ends up being 1?

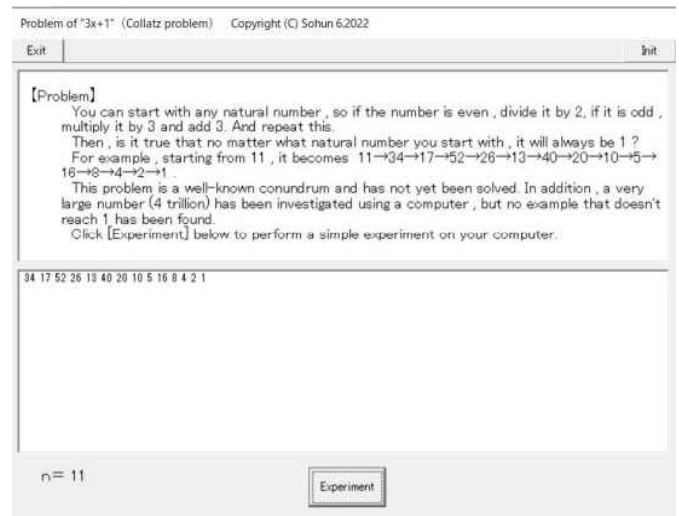
For example, if you start with 11, $11 \rightarrow 34 \rightarrow 17 \rightarrow 52 \rightarrow 26 \rightarrow 13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$.

This problem is a famous conundrum that has not yet been solved.

Also, very large numbers (4 trillion have been investigated using computers).

However, no example has been found where it does not become 1.

I tried a sample experiment using a computer.



(2) Experimental result (VB version simulation)

【Experiment day】

March 13 . 2024

【PC used】

Lavie NX850/N

【Software used】

Self-made software

『3x+1 problem 6』

【Method of operation】

Click the [Experiment] button.

Enter a natural number between 3 and 10,000,000 in half-width characters.

Click the [OK] button.

Starting with the natural number you input, the display shows how the number is divided by 2 if it is even, or multiplied by 3 and then add by 1 if it is odd, and repeated.

Click the [Init] button to restart the experiment from the beginning.

【Consideration】

The 1st experiment started at 17. $17 \rightarrow 52 \rightarrow 26 \rightarrow 13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$.

The 2nd experiment started at 111. $111 \rightarrow 334 \rightarrow 167 \rightarrow 502 \rightarrow 251 \rightarrow 754 \rightarrow 377 \rightarrow 1132 \rightarrow 566 \rightarrow 283 \rightarrow 850 \rightarrow 425 \rightarrow 1276 \rightarrow 638 \rightarrow 319 \rightarrow 958 \rightarrow 479 \rightarrow 1438 \rightarrow 719 \rightarrow 2158 \rightarrow 1079 \rightarrow 3238 \rightarrow 1619 \rightarrow 4858 \rightarrow 2429 \rightarrow 7288 \rightarrow 3644 \rightarrow 1822 \rightarrow 911 \rightarrow 2734 \rightarrow 1367 \rightarrow 4102 \rightarrow 2051 \rightarrow 6154 \rightarrow 3077 \rightarrow 9232 \rightarrow 4616 \rightarrow 2308 \rightarrow 1154 \rightarrow 577 \rightarrow 1732 \rightarrow 866 \rightarrow 433 \rightarrow 1300 \rightarrow 650 \rightarrow 325 \rightarrow 976 \rightarrow 488 \rightarrow 244 \rightarrow 122 \rightarrow 61 \rightarrow 184 \rightarrow 92 \rightarrow 46 \rightarrow 23 \rightarrow 70 \rightarrow 35 \rightarrow 106 \rightarrow 53 \rightarrow 160 \rightarrow 80 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$.

The 3rd experiment started at 11111. $11111 \rightarrow \dots \rightarrow 52 \rightarrow 26 \rightarrow 13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$.

The 4th experiment started at 1717171. $1717171 \rightarrow \dots \rightarrow 52 \rightarrow 26 \rightarrow 13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$.

The 5th experiment started at 9999999. $9999999 \rightarrow \dots \rightarrow 52 \rightarrow 26 \rightarrow 13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$.

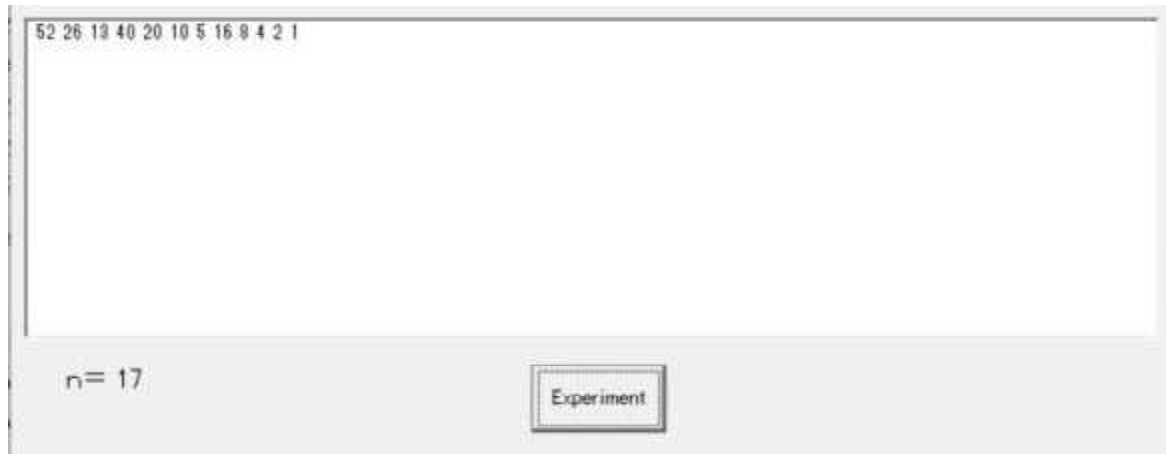
Modeling and Simulation III

3.13.2024
Sohun

5 Collatz Problem

(2) Experimental result (VB version simulation)

① 1st experiment



② 2nd experiment



Modeling and Simulation III

3.13.2024
Sohun

5 Collatz Problem

(2) Experimental result (VB version simulation)

③ 3rd experiment

```
33334 16667 50002 25001 75004 37502 18751 56254 28127 84382 42191 126574 63287 189862 94931 284794 142397 427192
213596 106798 53399 160198 80099 240298 120149 360448 180224 90112 45056 22528 11264 5632 2816 1408 704 352 176 88 44
22 11 34 17 52 26 13 40 20 10 5 16 8 4 2 1
```

n= 11111

Experiment

④ 4th experiment

```
5151514 2575757 7727272 3863636 1931818 965909 2897728 1448864 724432 362216 181108 90554 45277 135832 67916 33958
16979 50938 25469 76408 38204 19102 9551 28654 14327 42982 21491 64474 32237 96712 48356 24178 12089 36268 18134 9067
27202 13601 40804 20402 10201 30604 15302 7651 22954 11477 34432 17216 8608 4304 2152 1076 538 269 808 404 202 101 304
152 76 38 19 58 29 88 44 22 11 34 17 52 26 13 40 20 10 5 16 8 4 2 1
```

n= 1717171

Experiment

Modeling and Simulation III

3.13.2024
Sohun

5 Collatz Problem

(2) Experimental result (VB version simulation)

⑤ 5th experiment

```
29999998 14999999 44999998 22499999 67499998 33749999 101249998 50624999 151874998 75937499 227812498 113906249
341718748 170859374 85429687 256289062 128144531 384433594 192216797 576650392 288325196 144162598 72081299
216243898 108121949 324365848 162182924 81091462 40545731 121637194 60818597 182455792 91227896 45613948 22806974
11403487 34210462 17105231 51315694 25657847 76979542 38486771 115460314 57730157 173190472 86595236 43297618
21648889 64946428 32473214 16236607 48709822 24354911 73064734 36532367 109597102 54798551 184395654 82197827
248593482 123296741 369890224 184945112 92472556 46236279 23118139 69354418 34677209 104031628 52015814 26007907
78023722 39011861 117035584 58517792 29258896 14629448 7314724 3657362 1828681 5486044 2743022 1371511 4114534
2057267 6171802 3085901 9257704 4628852 2314426 1157213 3471640 1735820 867910 433955 1301866 650933 1952800 976400
488200 244100 122050 61025 183076 91538 45769 137308 68654 34327 102982 51491 154474 77237 231712 115856 57928 28964
14482 7241 21724 10862 5431 16294 8147 24442 12221 36664 18332 9166 4583 13750 6875 20626 10313 30940 15470 7735 23206
11603 34810 17405 52216 26108 13054 6527 19582 9791 29374 14687 44062 22031 66094 33047 99142 49571 148714 74357
223072 111536 55768 27884 13942 6971 20914 10457 31372 15686 7843 23530 11765 35296 17648 8824 4412 2206 1103 3310 1655
4966 2483 7450 3725 11176 5588 2794 1397 4192 2096 1048 524 262 131 394 197 592 296 148 74 37 112 56 28 14 7 22 11 34 17
52 26 13 40 20 10 5 16 8 4 2 1
```

n = 9999999

Experiment

Modeling and Simulation III

3.17.2024
Sohun

6 Pythagorean Numbers

(1) Experiment overview

Three positive integers (a, b, c) that satisfy the Pythagorean theorem are called Pythagorean triples.

To satisfy the Pythagorean theorem is to satisfy $a^2+b^2=c^2$.

When a is odd, the Pythagorean triples can be found by using

$$b=(a^2-1) \div 2$$

$$c=(a^2+1) \div 2 .$$

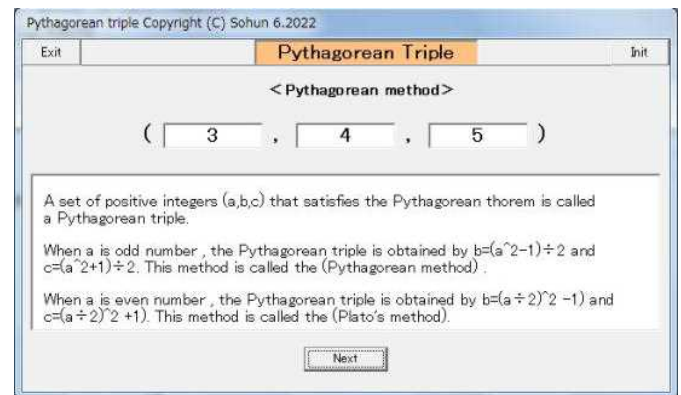
This method is called the Pythagorean method.

When a is even, the Pythagorean triples can be found by using

$$b=(a \div 2)^2-1$$

$$c=(a \div 2)^2+1.$$

This method is called Plato's method.



(2) Experimental result (VB version simulation)

【Experiment day】

March 17 . 2024

【PC used】

Lavie NX850/N

【Software used】

Self-made software

『Pythagorean number 6』

【Method of operation】

When you click the [Next] button, the value of integer a will increase by 1, and the values of integer b and integer c at that time will be displayed. (In this case, a, b, and c satisfy the Pythagorean theorem $a^2+b^2=c^2$).

Click the [Init] button to restart the experiment from a=3.

【Consideration】

The 1st experiment is when a=102. Since a=102 is an even number, We will use plato's method.

$$b=(a \div 2)^2-1=(102 \div 2)^2-1=2600$$

$$c=(a \div 2)^2+1=(102 \div 2)^2+1=2602$$

$$a^2=10404$$

$$b^2=6760000$$

$$c^2=6770404$$

$a^2+b^2=c^2$ is satisfied.

The 2nd experiment is when a=103. Since a=103 is an odd number. We will use Pythagorean method.

$$b=(a^2-1) \div 2=(103^2-1) \div 2=5304$$

$$c=(a^2+1) \div 2=(103^2+1) \div 2=5305$$

$$a^2=10609$$

$$b^2=28132416$$

$$c^2=28143025$$

$a^2+b^2=c^2$ is satisfied.

The 3rd experiment uses Plato's method, and the 4th experiment uses Pythagorean method.

In both cases, the Pythagorean theorem $a^2+b^2=c^2$ is satisfied.

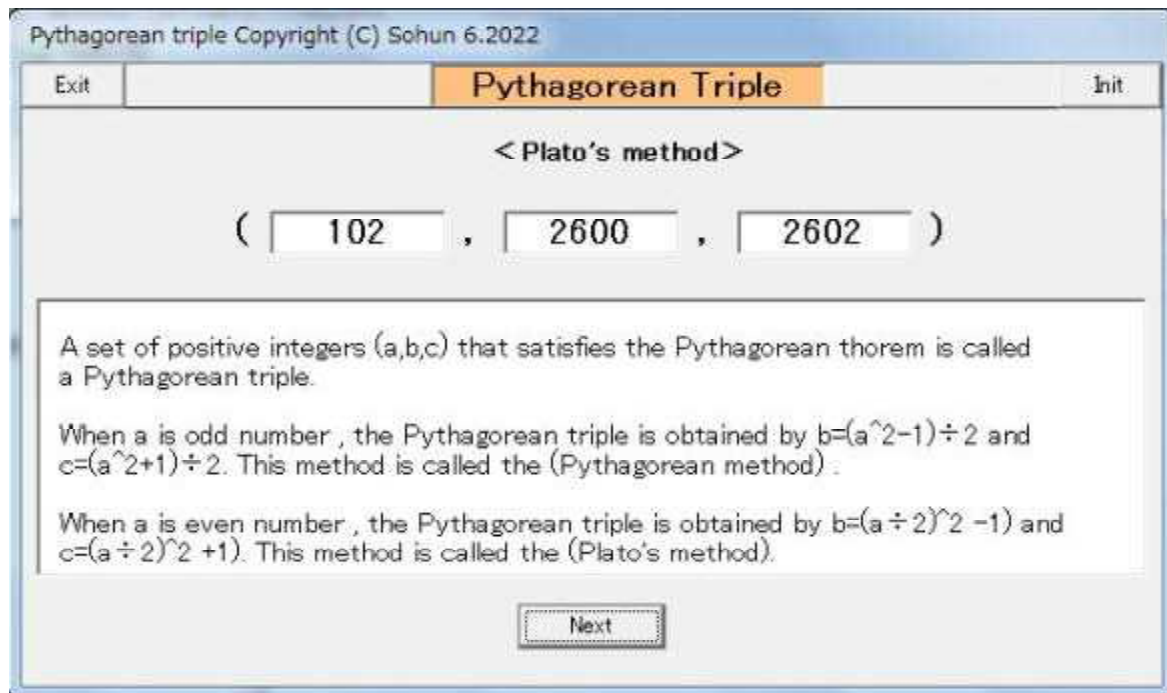
Modeling and Simulation III

3.17.2024
Sohun

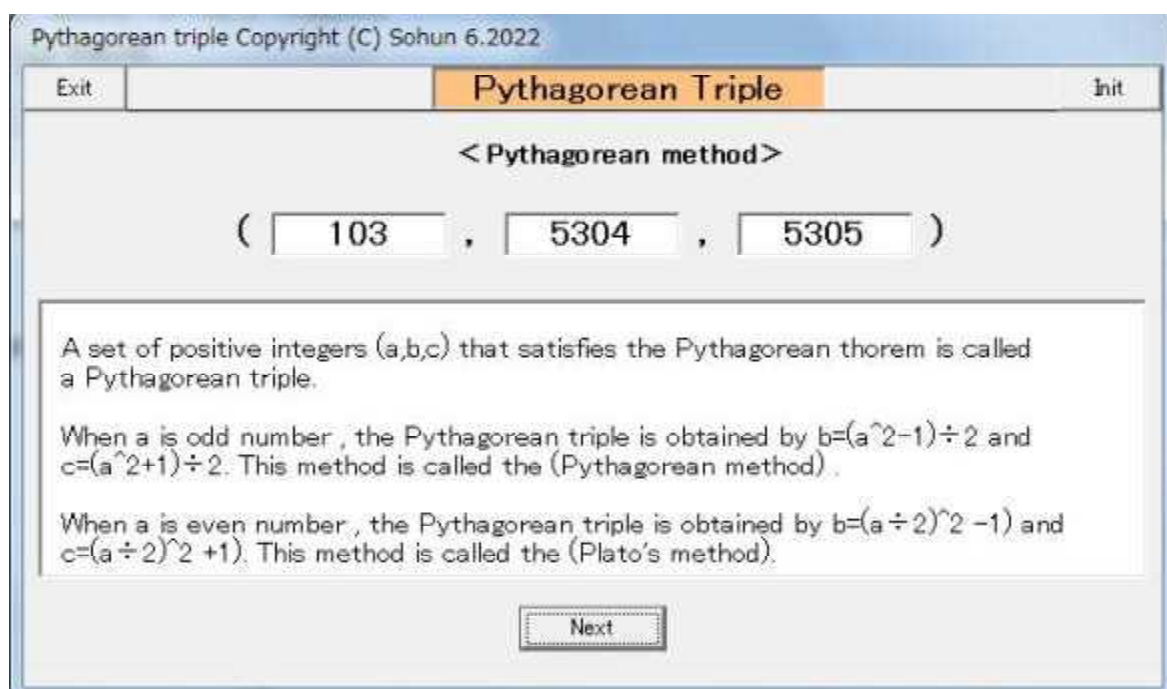
6 Pythagorean Numbers

(2) Experimental result (VB version simulation)

① 1st experiment



② 2nd experiment



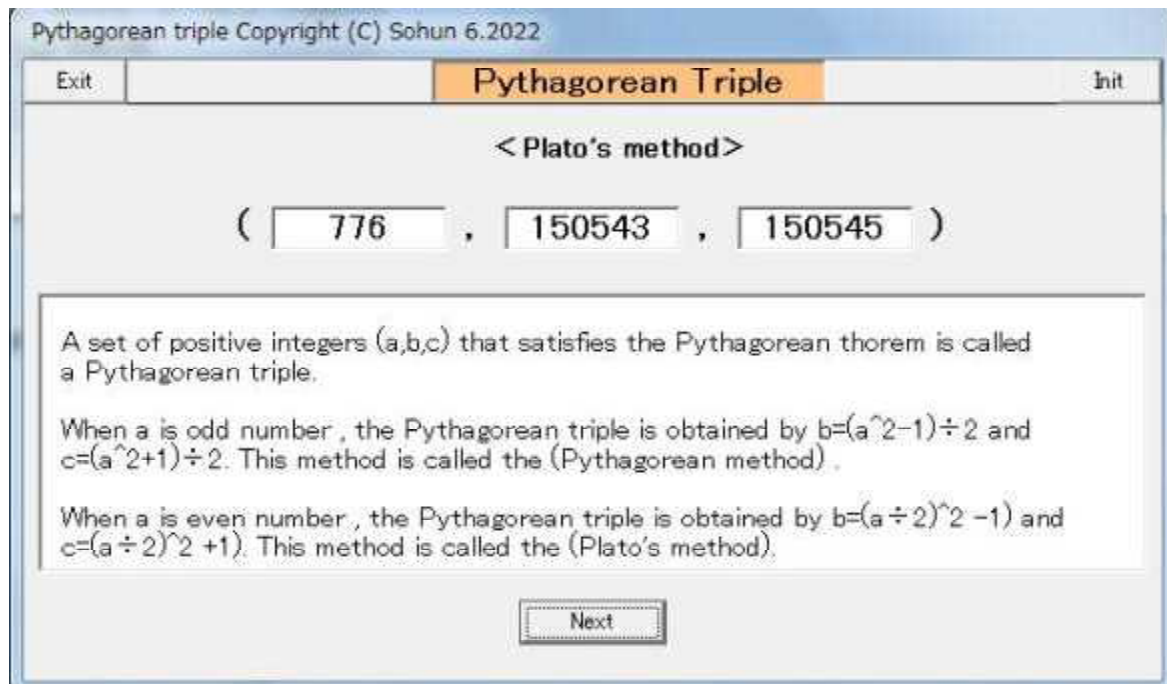
Modeling and Simulation III

3.17.2024
Sohun

6 Pythagorean Numbers

(2) Experimental result (VB version simulation)

③ 3rd experiment



④ 4th experiment

