Interesting Simulation Ⅰ（Grapes）

2024.1.27
Sohun

1　Intersection of circle and straight line

（１） Exam question 1

On a coordinate plane with O as the origin , there is a circle C : $x^2+y^2+2x-6y=0$ and a straight line L : $3x-y+k=0$ (k is a constant). The circle C and the straight line L intersect at two different points P and Q.

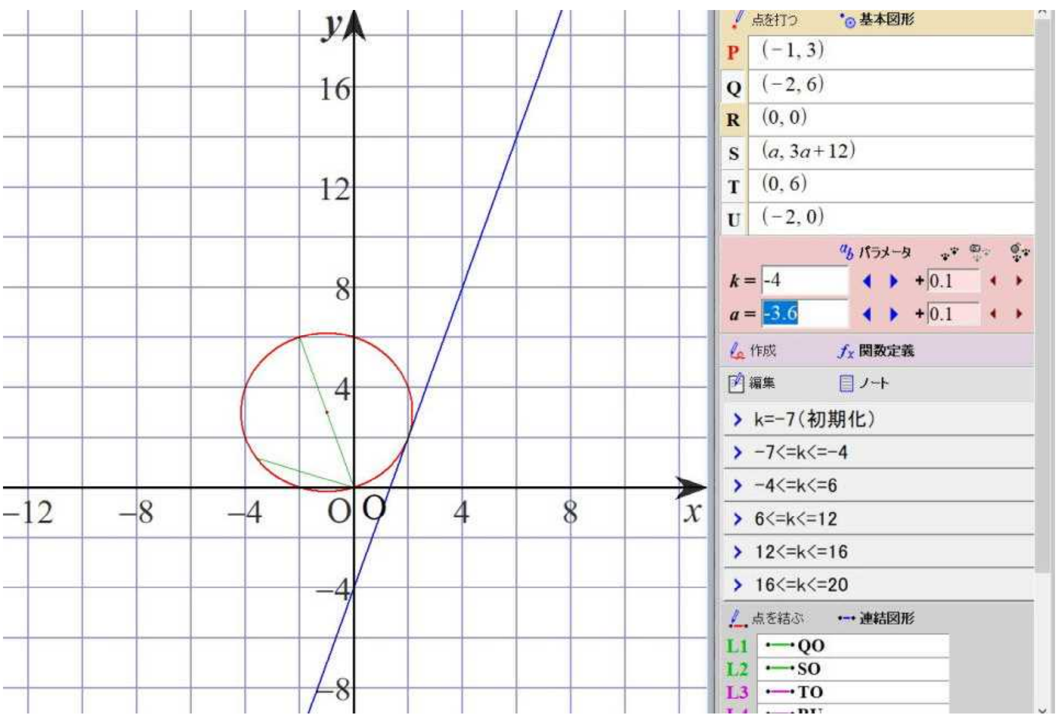
① Find the range of possible values for k.
② Find the value of k when △OPQ is a right triangle.

（２） Experimental result（**Grapes** version simulation）

【Experiment day】
January 27 , 2024
【PC used】
Lavie NX850 / N
【GRAPES used】
GRAPES 7.84
【Script used】
Self-made file
『examquestion1.gps』

【Consideration】
I varied the value of k from -7 to 20 and observed the common points between circle C and straight line L. When k=-4 and k=16 , straight line L touches circle C.（Since the distance between the center of the circle C and the straight line L is equal to the radius of the circle , k=-4 and 16 can be found.）
When k=6 , the hypotenuse PQ of △OPQ becomes the diametor of the circle C , so △OPQ becomes a right triangle. When k=12 , the hypotenuse OP（or the hypotenuse OQ） of △OPQ becomes the diameter of the circle C , so △OPQ becomes a right triangle.
When -4<k<16 , line L intersects circle C at two different points. Therefore , the range of values of k where the circle C and the straight line L intersect at two different points is -4<k<16.
The value of k when △OPQ becomes a right triangle is k=6,12.

① When the value of k is -4



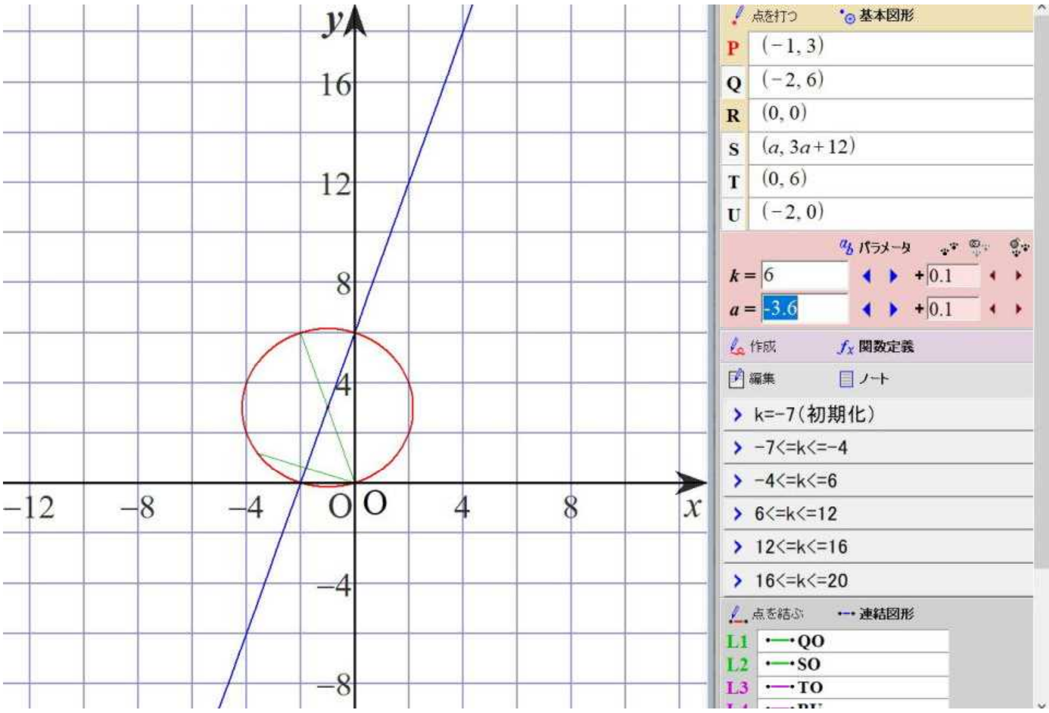
Interesting Simulation Ⅰ（Grapes）

2024.1.27
Sohun

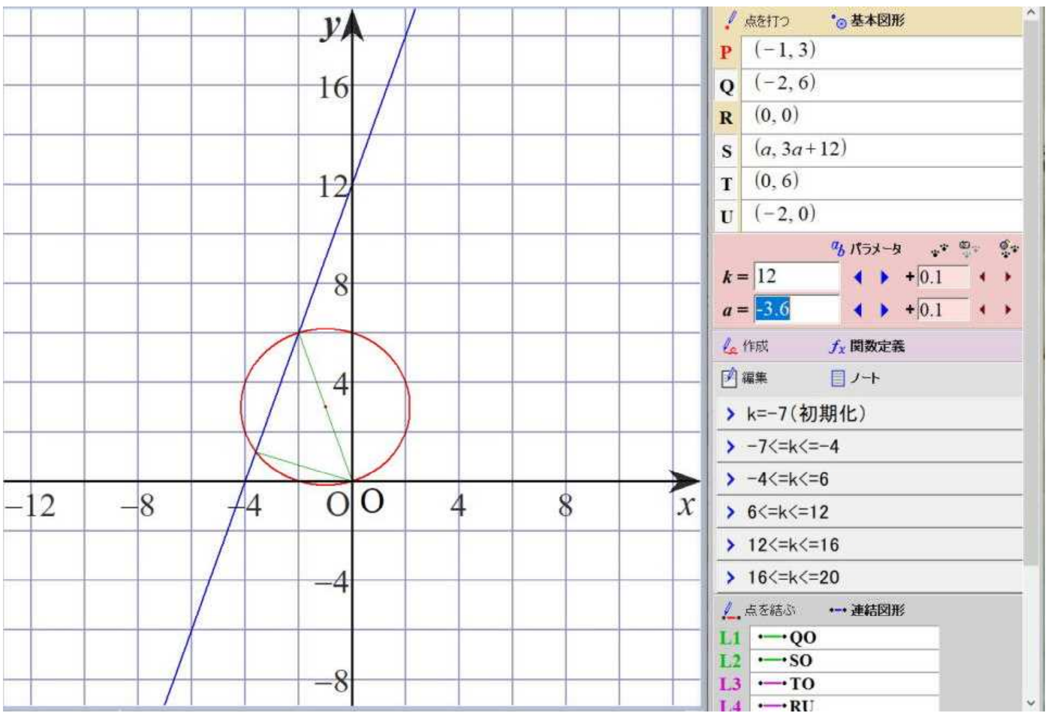
1　Intersection of circle and straight line

（２）Experimental result（Grapes version simulation）

② When the value of k is 6



③ When the value of k is 12



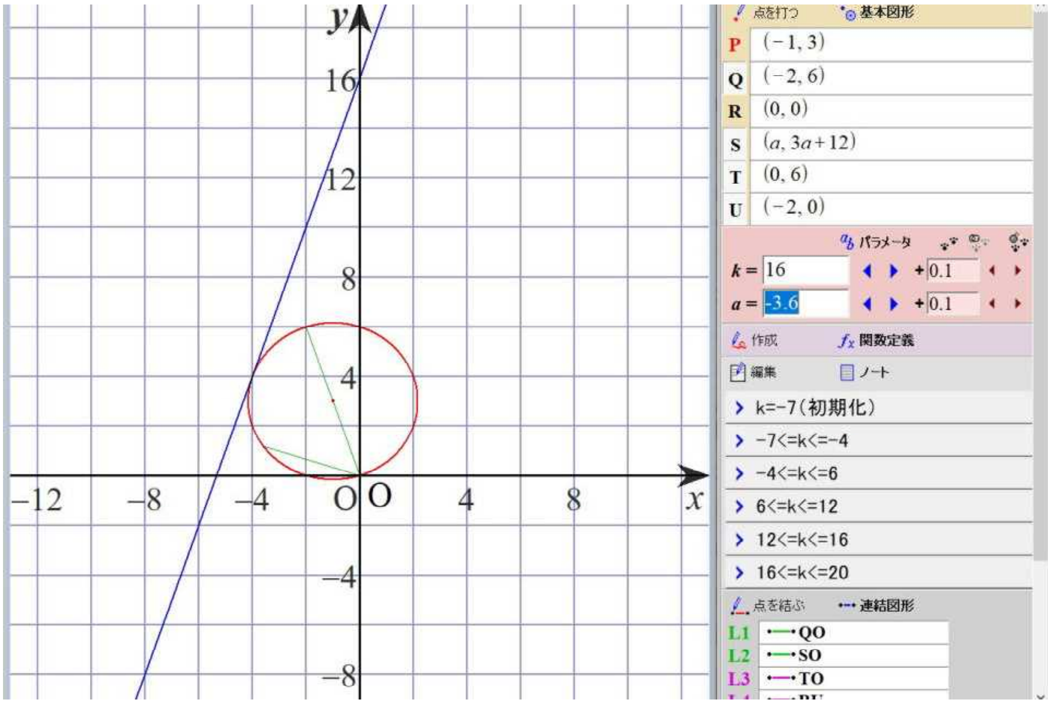
Interesting Simulation Ⅰ（Grapes）

2024.1.27
Sohun

1　Intersection of circle and straight line

（２）Experimental result（Grapes version simulation）

④ When the value of k is 16



Interesting Simulation Ⅰ（Grapes）

2024.1.29
Sohun

2　Intersection of two circles

（１） Exam question 2

On the coordinate plane , there is a circle C1 whose diameter is at both ends of two points A (3,4) and B (5,8) . There is also a circle C2 : $x^2+y^2-4ax-2ay+5a^2-5=0$.

However , a is a constant.

① Find the equation of C1.

② Find the range of values of a such that C1 and C2 intersect at two different points.

（２） Experimental result （Grapes version simulation）

【Experiment day】
January 29 , 2024
【PC used】
Lavie NX850 / N
【GRAPES used】
GRAPES 7.84
【Script used】
Self-made file
『examquestion2.gps』

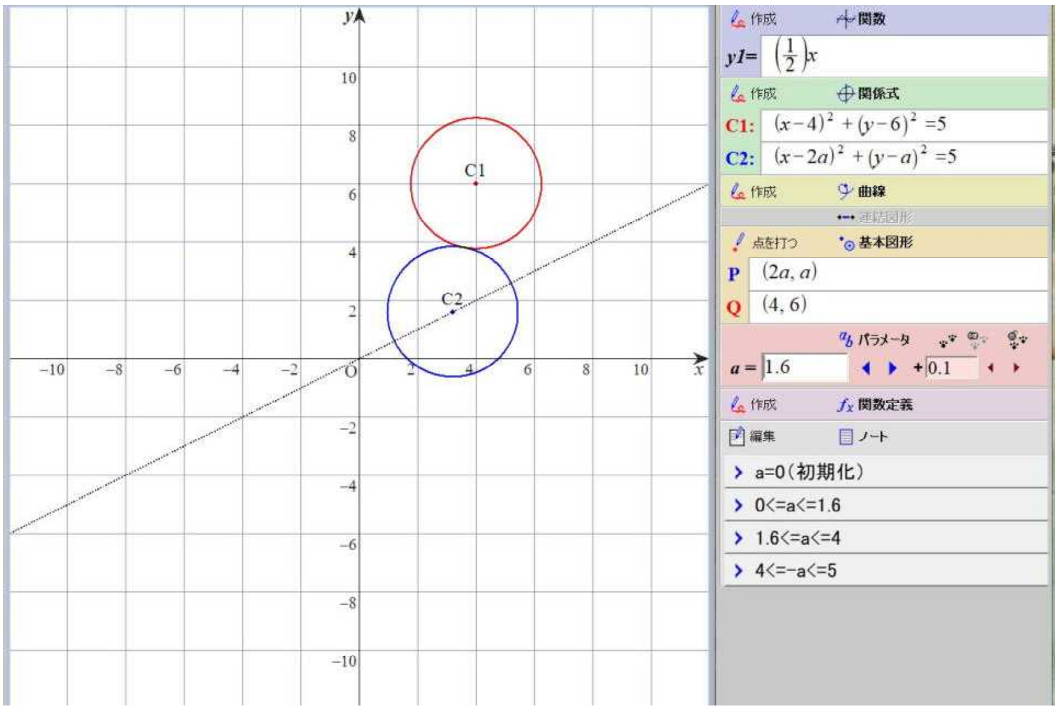
【Consideration】

I varied the value of a from 0 to 5 and observed the common points between circles C1 and C2. When a=1.6 , circle C2 touches circle C1. When $1.6<a<4$, circle C2 and circle C1 intersect at two different points. When a=4 , circle C2 touches circle C1.

（When the circle C1 and the circle C2 are circumscribed , the distance between the centers of the circle C1 and C2 is equal to the sum of the radius of C1 and the radius of C2 , so a=1.6 , 4 can be calculated.）

Therefore , the range of values of a where the circles C1 and C2 intersect at two different points is $1.6<a<4$.

① When the value of a is 1.6



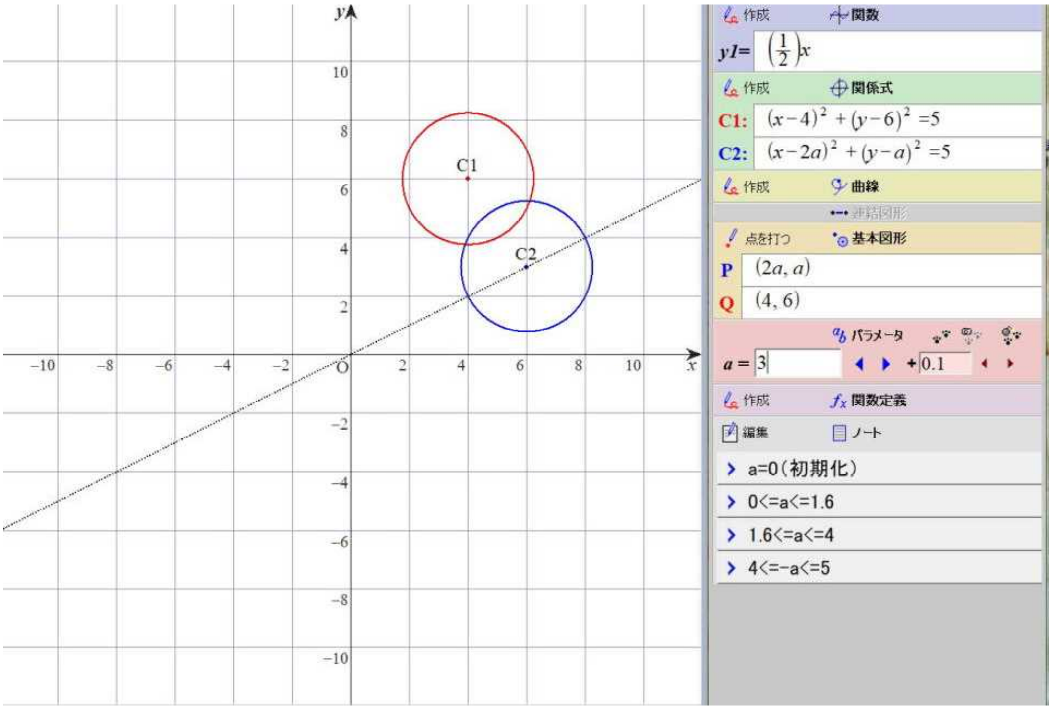
Interesting Simulation Ⅰ（Grapes）

2024.1.29
Sohun

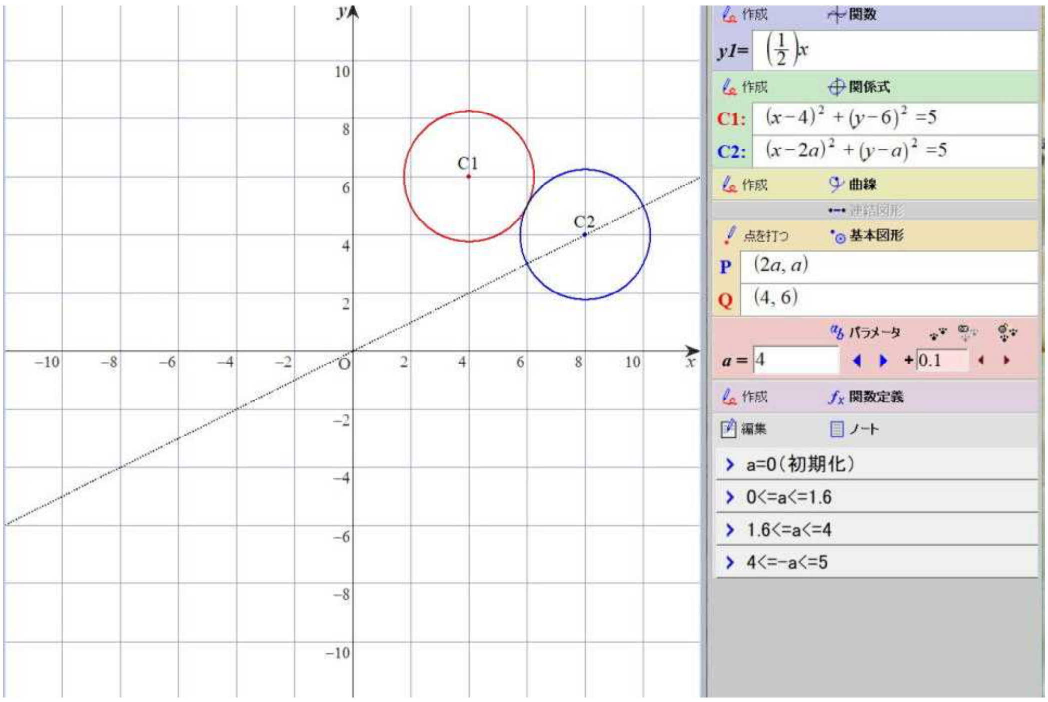
2　Intersection of two circles

（２）Experimental result（Grapes version simulation）

② When the value of a is 3



③ When the value of a is 4



Interesting Simulation Ⅰ（Grapes）

2024.1.29
Sohun

3　Common points between circles

（１） Exam question 3

Find the range of the values of k such that the two circles $x^2 + y^2 = k^2$ (k>0)・・・①、$x^2 + y^2 - 8x - 4y + 15 = 0$・・・② have common points.

（２） Experimental result （**Grapes** version simulation）

【Experiment day】
January 29 , 2024
【PC used】
Lavie NX850 / N
【GRAPES used】
GRAPES 7.84
【Script used】
Self-made file
『examquestion3.gps』

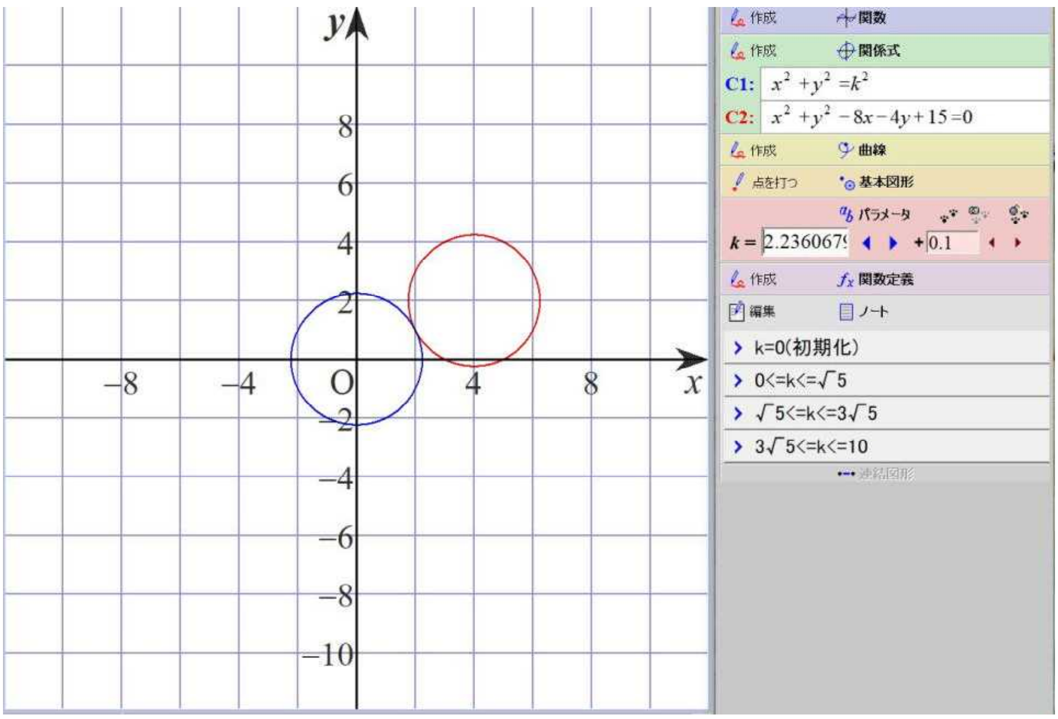
【Consideration】

I varied the value of k from 0 to 10 and observed the common points between circles ① and ②. When $k=\sqrt{5}$, circle ① and circle ② are circumscribed. When $\sqrt{5}<k<3\sqrt{5}$, circle ① and ② intersect at two different points. When $k=3\sqrt{5}$, circle ② is inscribed in circle ①.

（From the relationship between the distance between the centers of circles ① and ② and radii of circles ① and ② , the values of $k=\sqrt{5}$, $3\sqrt{5}$ where they are circumscribed and inscribed can be calculayed.）

Therefore , the range of values of k in which circle ① and ② have common points is $\sqrt{5} \leqq k \leqq 3\sqrt{5}$.

① When the value of k is $\sqrt{5}$



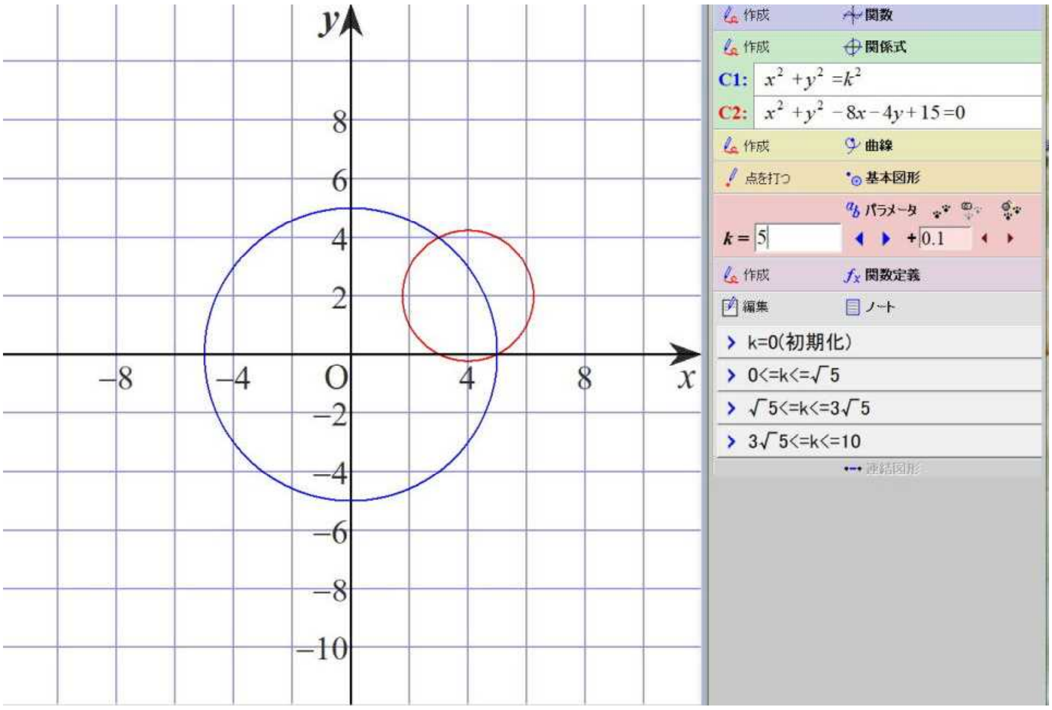
Interesting Simulation Ⅰ（Grapes）

2024.1.29
Sohun

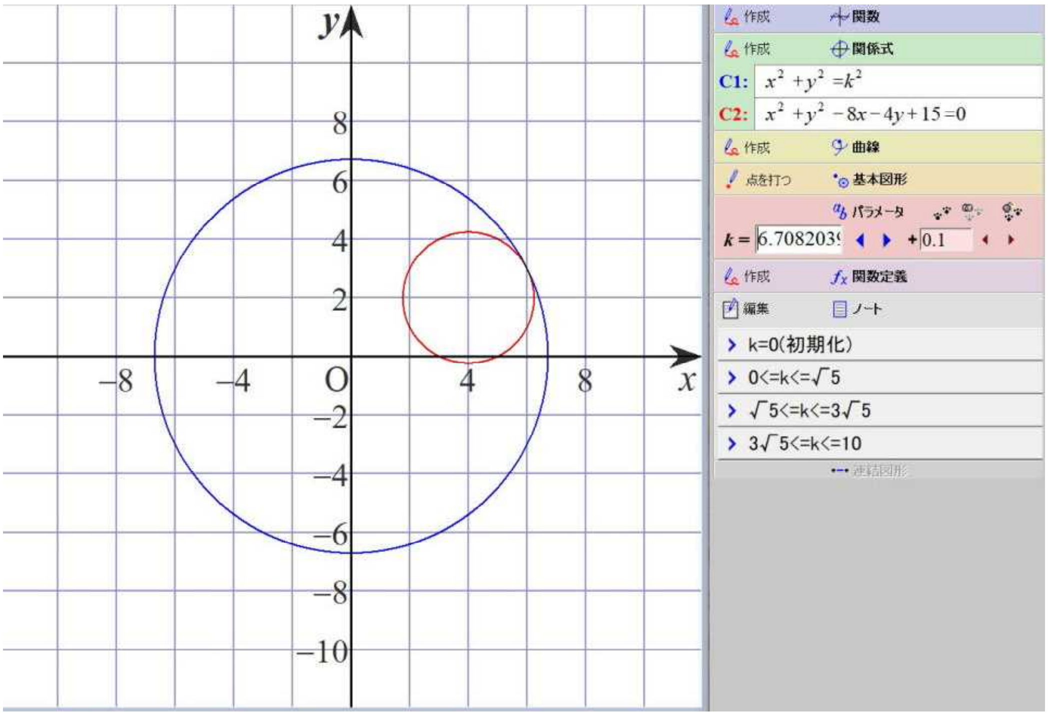
３　Common points between circles

（２）Experimental result（Grapes version simulation）

② When the value of k is 5



③ When the value of k is 3 √ 5



Interesting Simulation Ⅰ（Grapes）

2024.1.30
Sohun

4　A straight line passing through the intersection of two circles

（１）Exam question 4

Find the equation of the straight line that passes through the intersection of the two circles ①: $x^2 + y^2 = 5$, ②: $(x-1)^2 + (y-2)^2 = 4$

（２）Experimental result（**Grapes** version simulation）

【Experiment day】
January 30 , 2024
【PC used】
Lavie NX850 / N
【GRAPES used】
GRAPES 7.84
【Script used】
Self-made file
『examquestion4.gps』

【Consideration】

The equation of the figure passing through the intersection of circles ① and ② can be expressed as $(x-1)^2+(y-2)^2-4+k(x^2+y^2-5)=0$・・・③. I varied the value of k from -10 to 20 and observed figure ③ passing through the intersections of circles ① and ②.

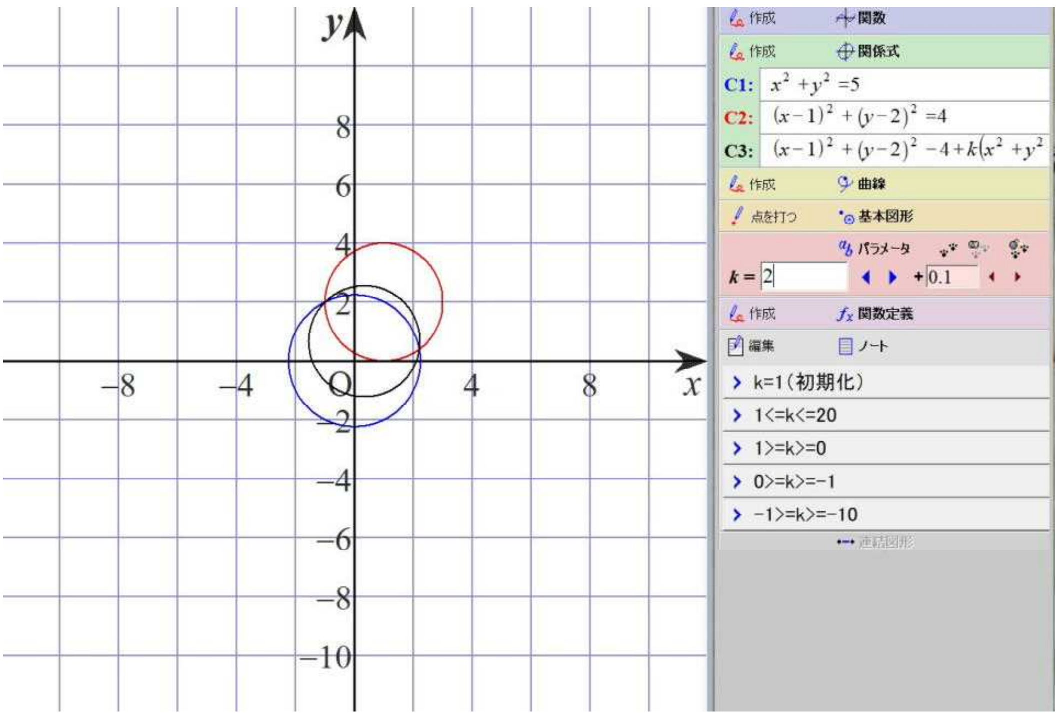
When k = -1 , figure ③ passing through the intersection of circle ① and circle ② becomes a straight line .

When k ≠ -1 , figure ③ passing through the intersection of circle ① and circle ② is a circle.

Therefore , the equation of the straight line passing through the intersection of circles ① and ② can be found by substituting -1 for k in equation ③.

$x+2y-3=0$

① When the value of k is 2



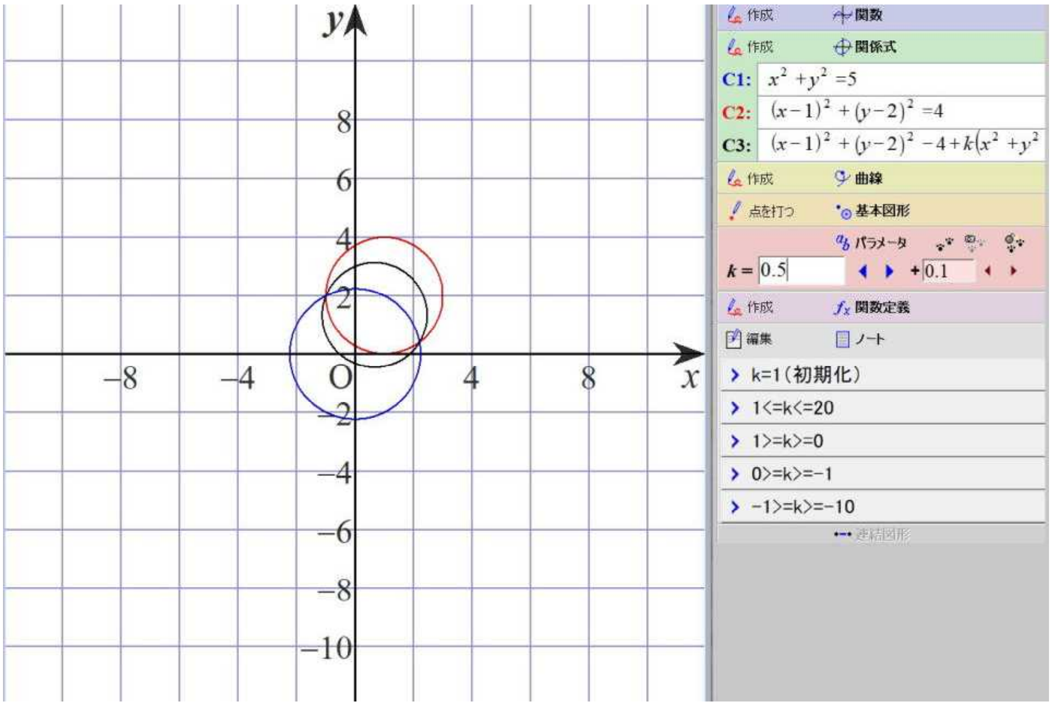
Interesting Simulation Ⅰ（Grapes）

2024.1.30
Sohun

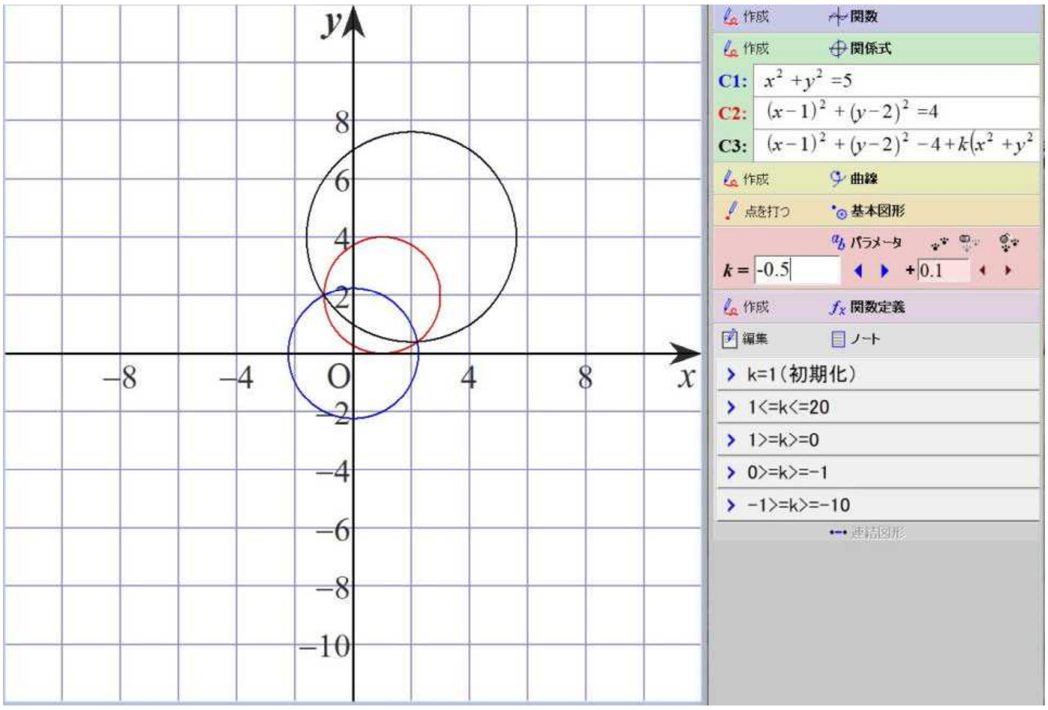
4　A straight line passing through the intersection of two circles

（２）Experimental result（Grapes version simulation）

② When the value of k is 0.5



③ When the value of k is -0.5



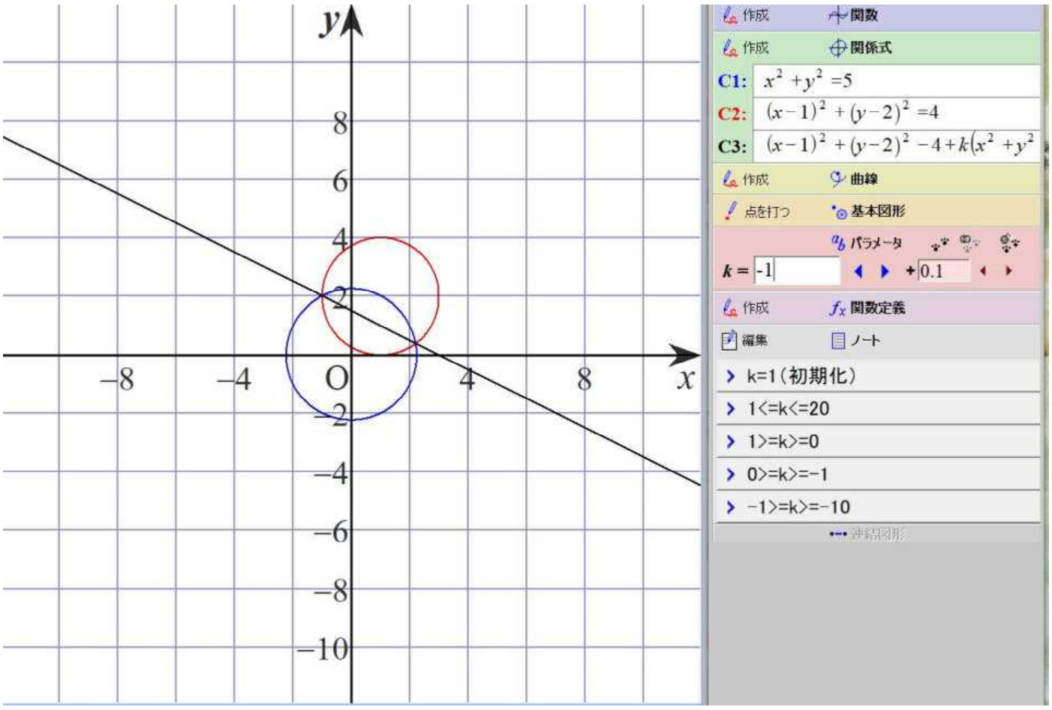
Interesting Simulation Ⅰ（Grapes）

2024.1.30
Sohun

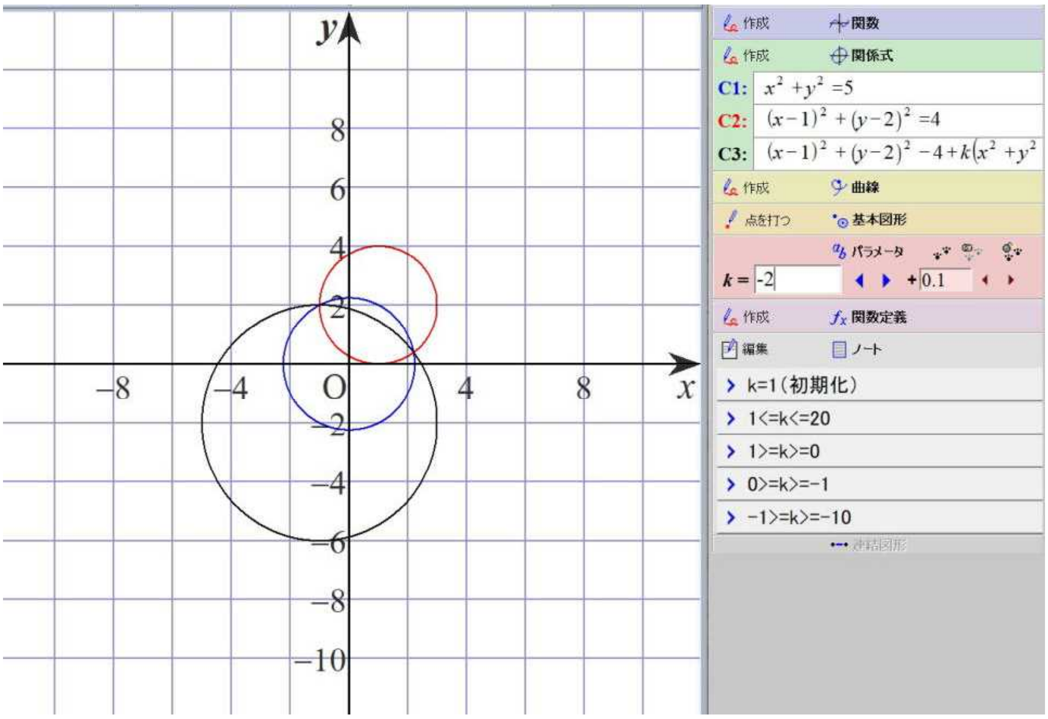
4　A straight line passing through the intersection of two circles

（２）Experimental result（Grapes version simulation）

④ When the value of k is -1



⑤ When the value of k is -2



Interesting Simulation Ⅰ（Grapes）

2024.1.30
Sohun

5　A straight line passing through the intersection of two straight lines

（１）Exam question 5

Find the equation of the straight line that passes through the intersection of the two straight lines ①: $2x+3y=7$ and ②: $4x+11y=19$, and passes through the point（5,4）.

（２）Experimental result（**Grapes** version simulation）

【Experiment day】
January 30 , 2024
【PC used】
Lavie NX850 / N
【GRAPES used】
GRAPES 7.84
【Script used】
Self-made file
『examquestion5.gps』

【Consideration】

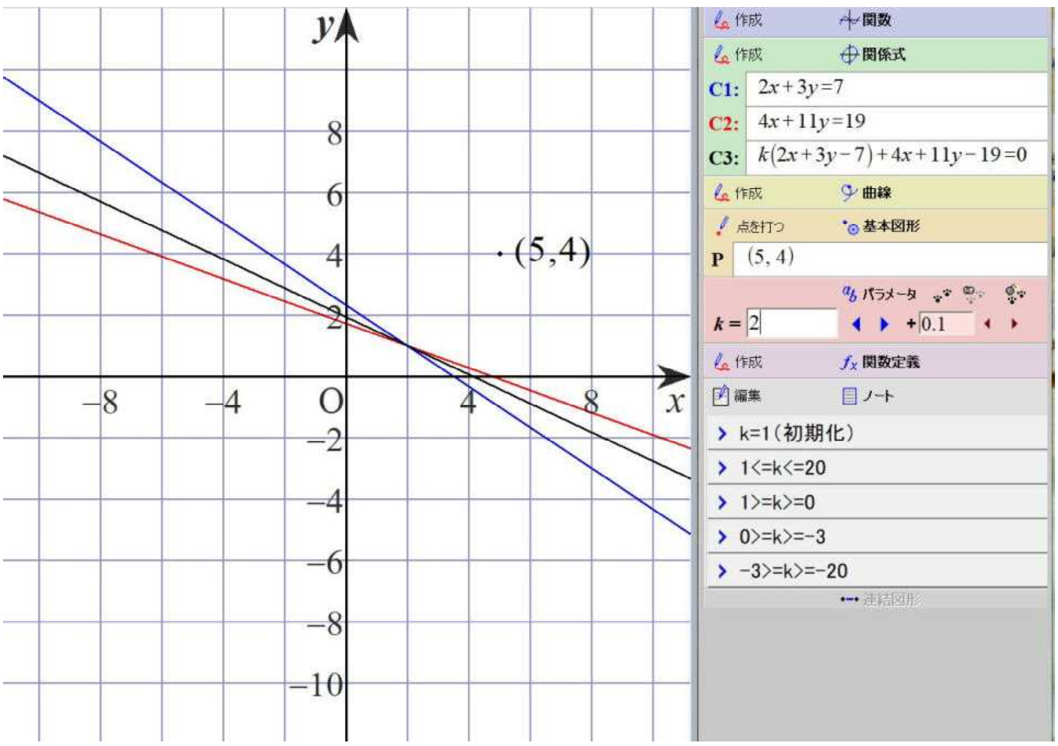
The equation of the straight line passing through the intersection of two straight lines ① and ① can be expressed as ③ : $k(2x+3y-7)+4x+11y-19=0$. The value of k was varied from -20 to 20 and a straight line ③ passing through the intersection of straight lines ① and ② was observed. When k = -3 , straight line ③ passes through the point（5,4）. When k ≠ -3 , straight line ③ doesn't pass through the point（5,4）.

Therefore , find the equation of the straight line that passes through the intersection of lines ① and ② , and passes through the point（5,4）by abstituting -3 for k in equation ③.

$x-y-1=0$.

（Substituting the coordinates of the passing point（5,4）, x=5 , y=4 into equation ③ , you will find k = -3 ）

① When the value of k is 2



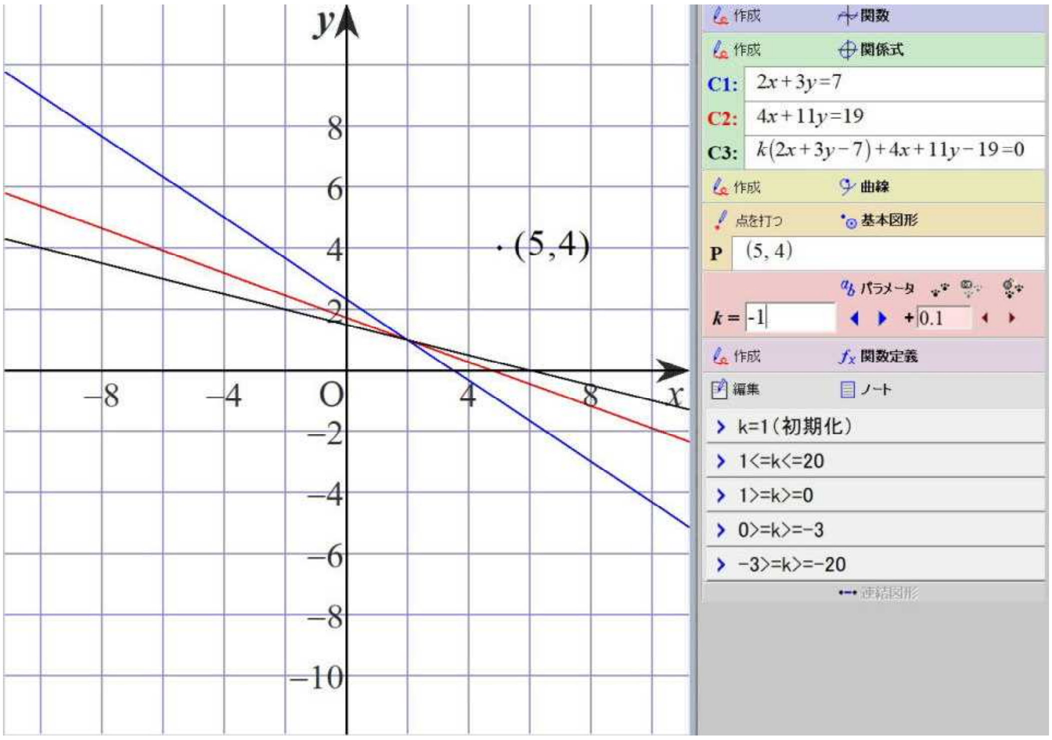
Interesting Simulation Ⅰ（Grapes）

2024.1.30
Sohun

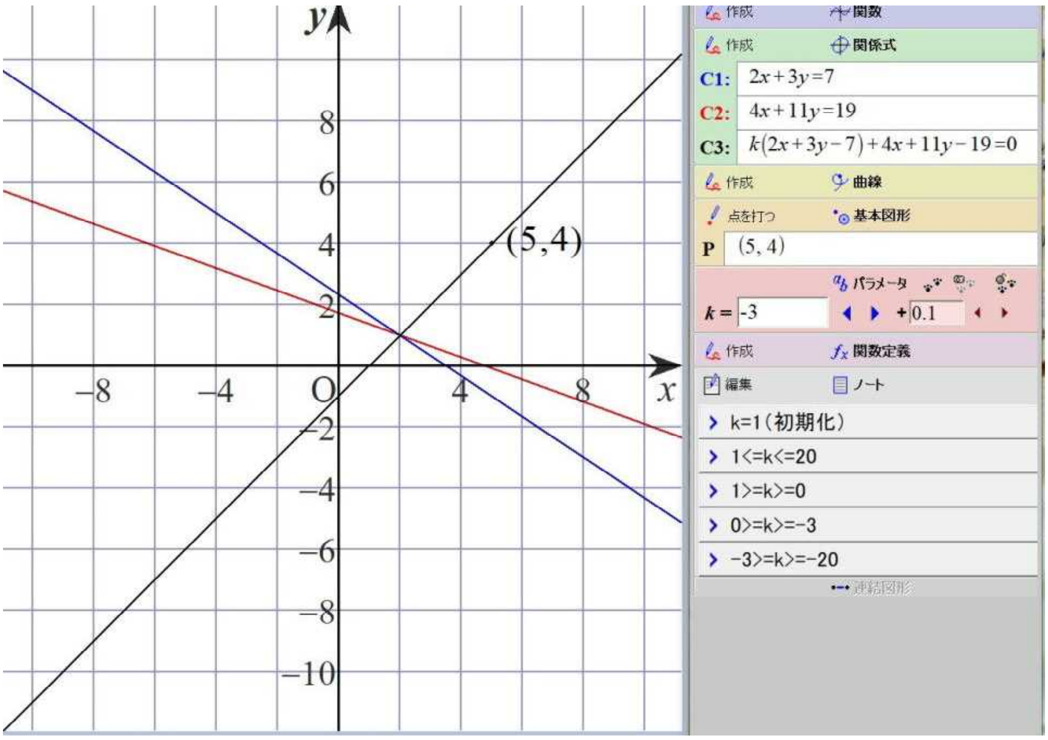
5　A straight line passing through the intersection of two straight lines

（２）Experimental result（Grapes version simulation）

② When the value of k is -1



③ When the value of k is -3



Interesting Simulation Ⅰ（Grapes）

2024.1.31
Sohun

6　Common points of circle and parabola

（１）Exam question 6

Find the value of a when circle ①: $x^2+y^2=1$ and parabola ②: $y=ax^2-2$ share only two different points.

（２）Experimental result（Grapes version simulation）

【Experiment day】
January 31 , 2024
【PC used】
Lavie NX850 / N
【GRAPES used】
GRAPES 7.84
【Script used】
Self-made file
『examquestion6.gps』

【Consideration】

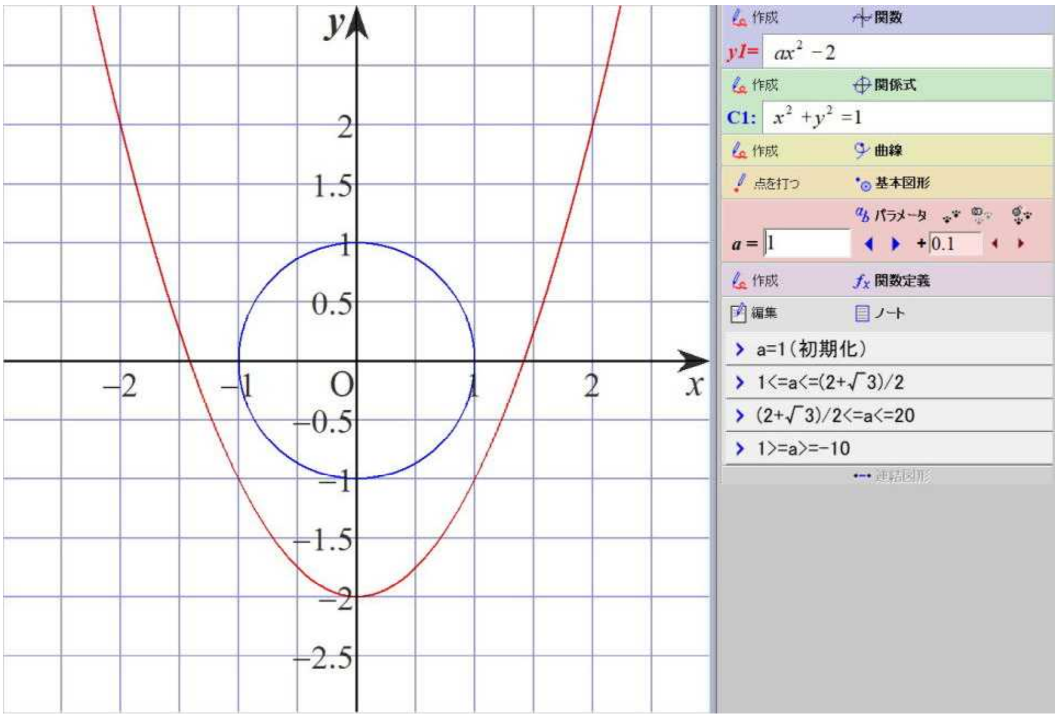
I varied the value of a from -10 to 20 and observed the common point between circle ① and parabola ②.

When a = $(2+\sqrt{3})/2$, circle ① and parabola ② share only two different points.

（If you combine ① and ② and calcurate using the multiple solution condition , you will find a = $(2\pm\sqrt{3})/2$. Moreover , a>0.25 can also be found from the condition that the answer is positive.）

Therefore , a=$(2+\sqrt{3})/2$

① When the value of a is 1



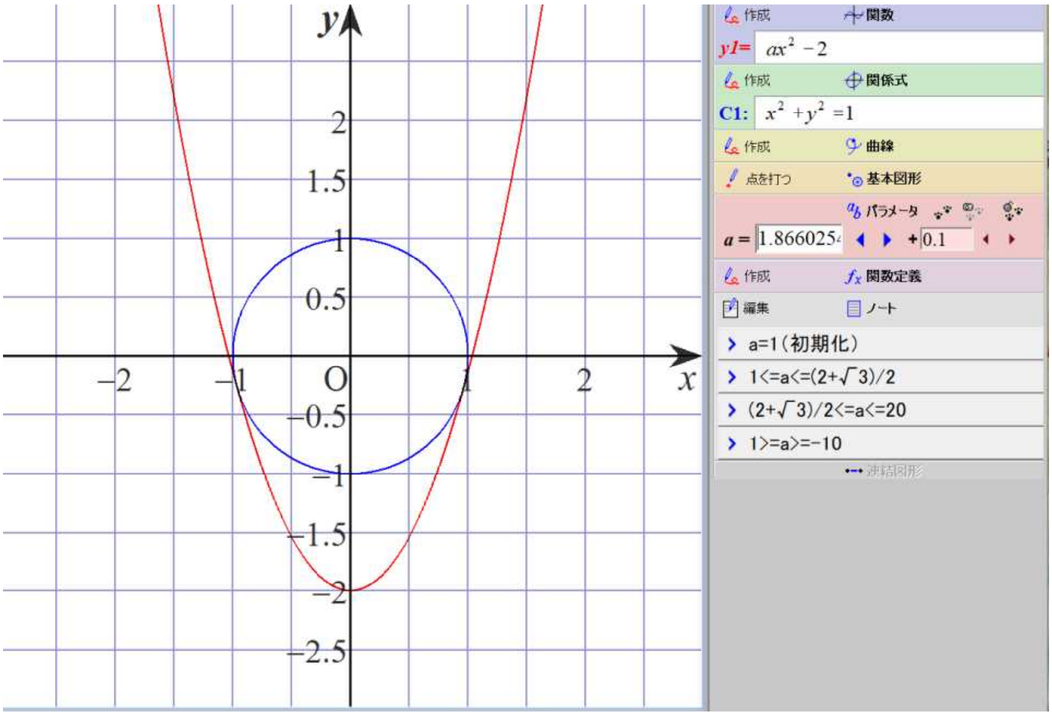
Interesting Simulation Ⅰ（Grapes）

2024.1.31
Sohun

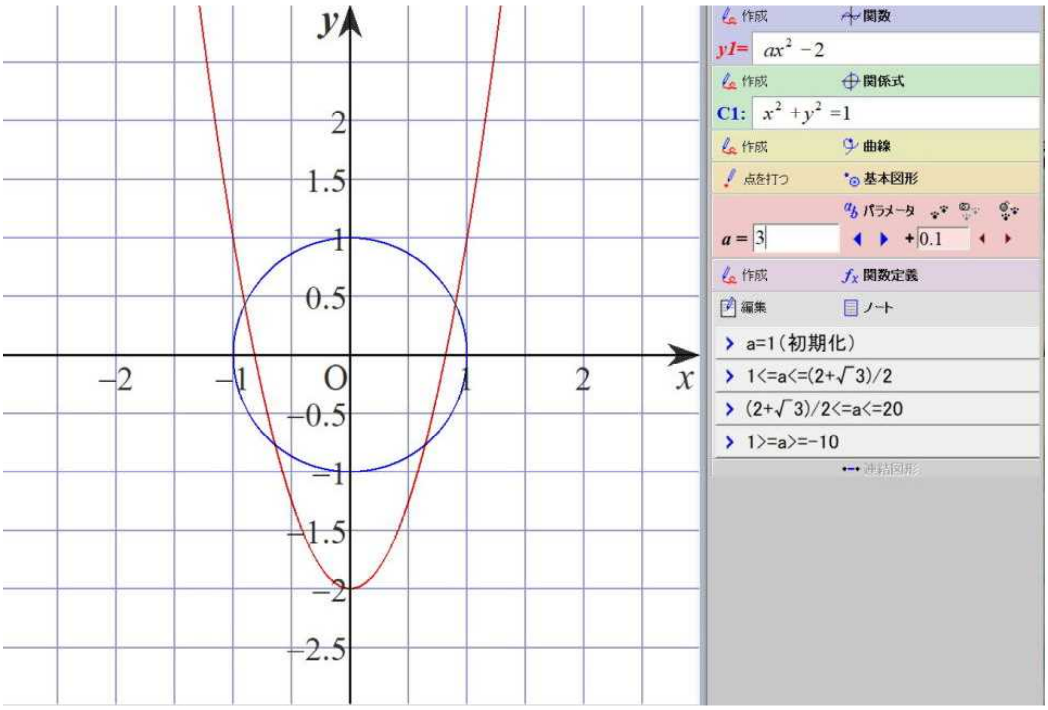
６　Common points of circle and parabola

（２）Experimental result（Grapes version simulation）

② When the value of a is（2+√ 3）/2



③ When the value of a is 3



Interesting Simulation Ⅰ（Grapes）

2024.2.1
Sohun

7　Positional relationship between two circles

（１）Exam question 7

Circle C1:（$(x-5)^2+(y-2)^2=16$

Circle C2:（$(x-1)^2+(y+1)^2=a+2$

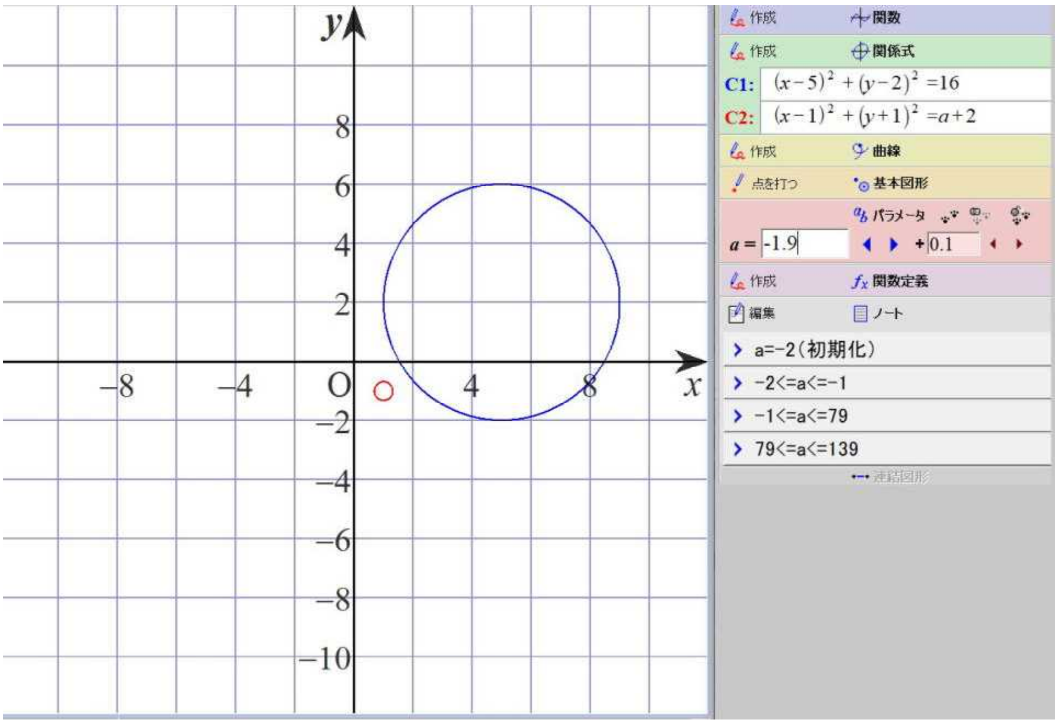
① Find the range of values of a when they are outside each other.
② Find the range of values of a when one is inside the other.

（２）Experimental result（**Grapes** version simulation）

【Experiment day】
February 1 , 2024
【PC used】
Lavie NX850 / N
【GRAPES used】
GRAPES 7.84
【Script used】
Self-made file
『examquestion7.gps』

【Consideration】
I varied the value of a from -2 to 139 and observed the positional relationship between circle C1 and circle C2. When $-2<a<-1$, circle C1 and circle C2 are outside each other. When a = -1 , circle C1 and circle C2 are circumscribed. When $-1<a<79$, circle C1 and circle C2 intersect at two different points. When a = 79 , circle C1 is inscribed in circle C2.
When $a>79$, circle C1 is inside circle C2.
Therefore , the range of values of a when they are outside each other is $-2<a<-1$. The range of values of a when one is inside the other is $a>79$.
（From the relationship between the distance between centers and their radiii , the values of a -1 and 79 when they touch are found.）

① When the value of a is -1.9



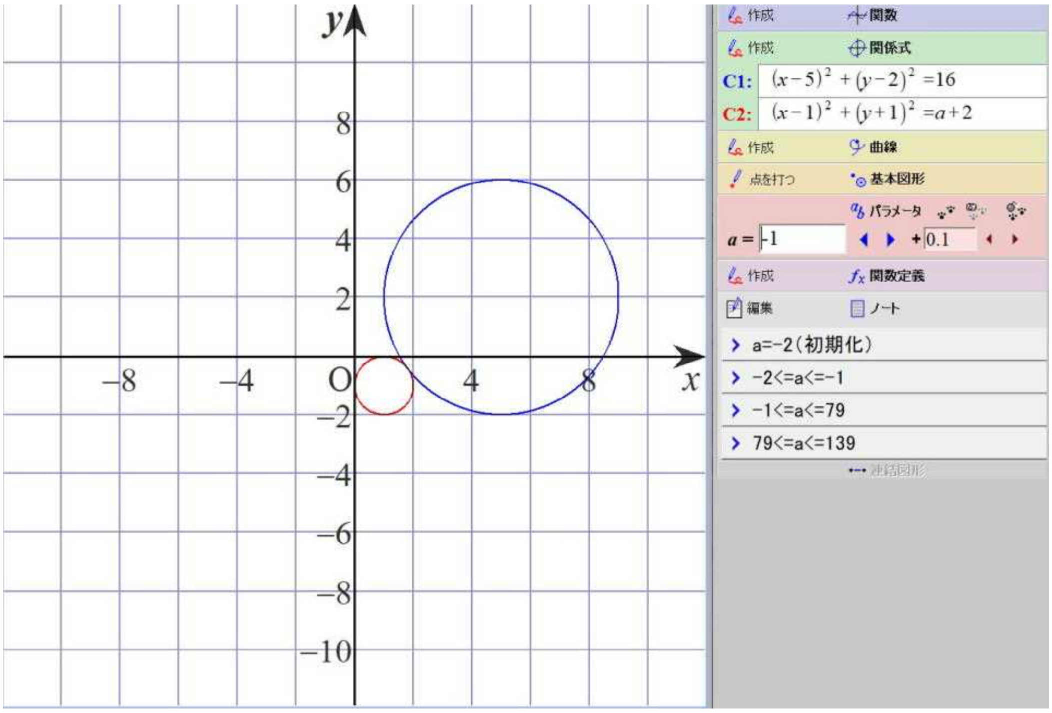
Interesting Simulation Ⅰ（Grapes）

2024.2.1
Sohun

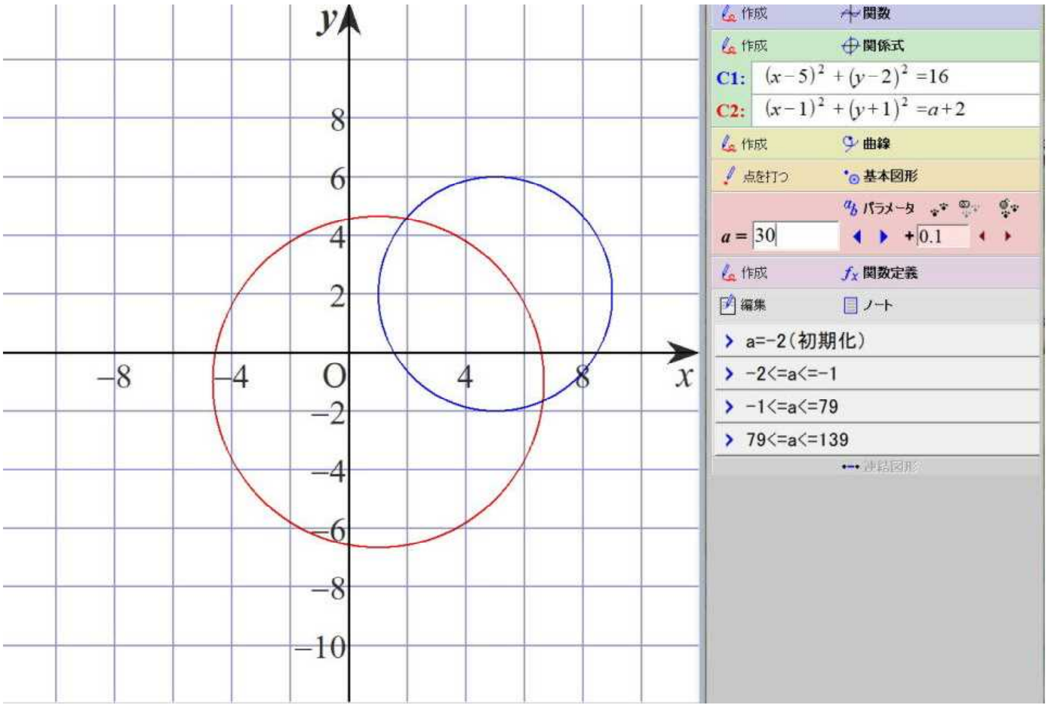
7　Positional relationship between two circles

（２）Experimental result（Grapes version simulation）

② When the value of a is -1



③ When the value of a is 30



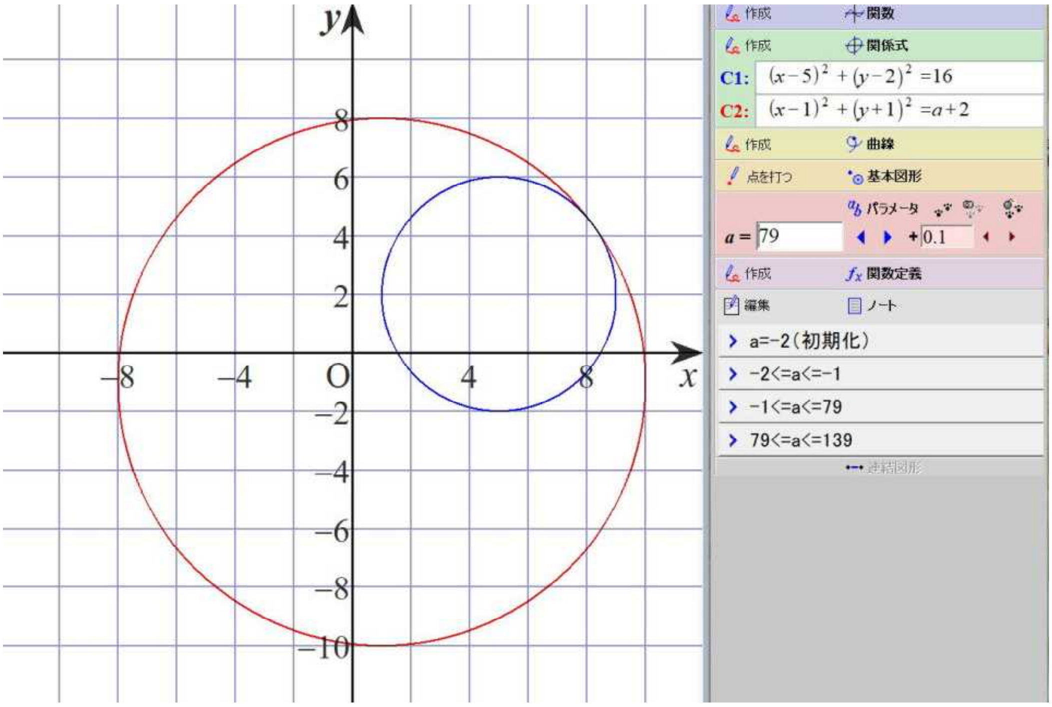
Interesting Simulation Ⅰ（Grapes）

2024.2.1
Sohun

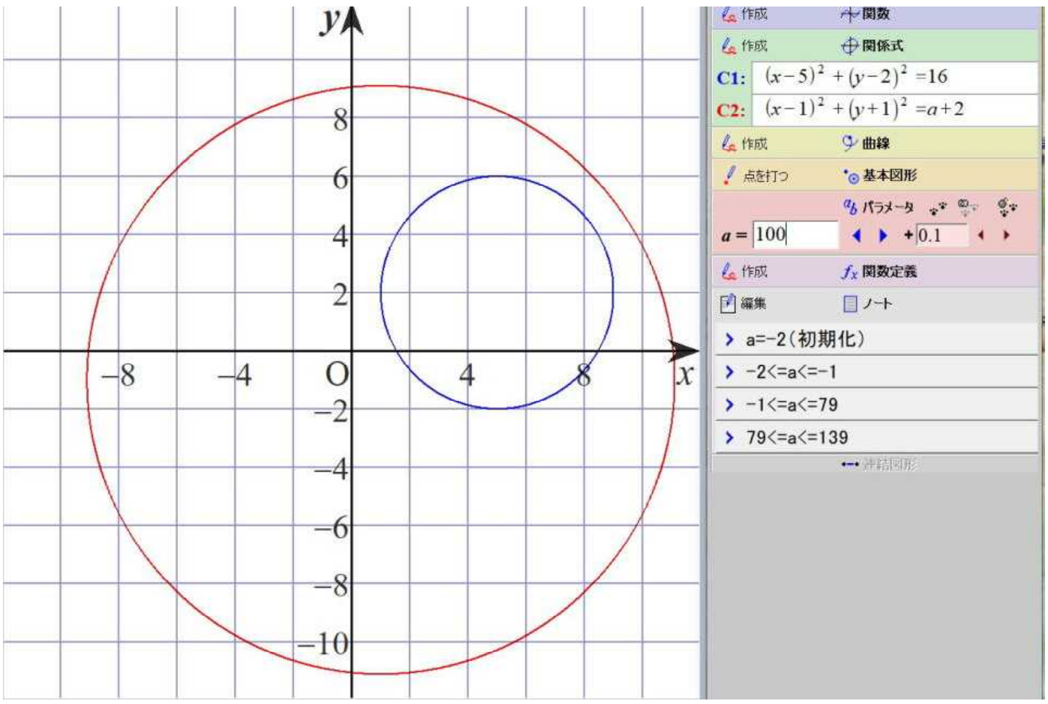
7　Positional relationship between two circles

（２）Experimental result（Grapes version simulation）

④ When the value of a is 79



⑤ When the value of a is 100



Interesting Simulation Ⅰ（Grapes）

2024.2.2
Sohun

8　Locus of the vertex of a parabola

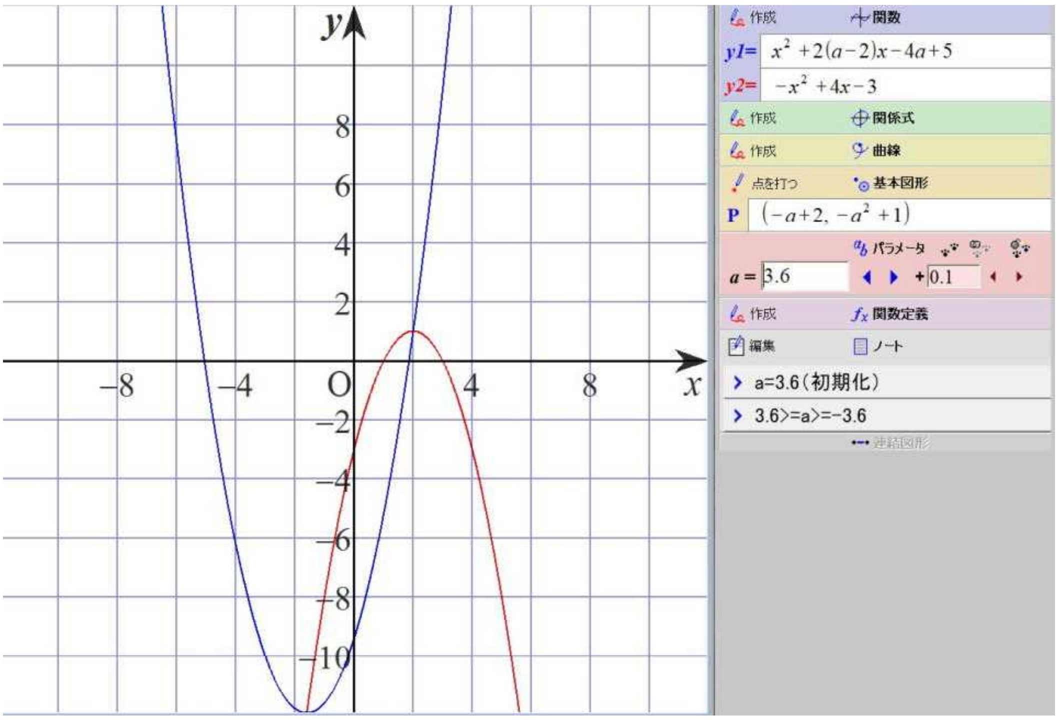
（１） Exam question 8
About parabola ①: $y=x^2+2(a-2)x-4a+5$
When a changes , the vertex of parabola ① draws a single curve.
Find the equation of this curve.

（２） Experimental result （Grapes version simulation）

【Experiment day】
February 2 , 2024
【PC used】
Lavie NX850 / N
【GRAPES used】
GRAPES 7.84
【Script used】
Self-made file
『examquestion8.gps』

【Consideration】
I varied the value of a from 3.6 to -3.6 and observed the locus of the vertex of parabola ①.
The vertex of parabola ① has moved on the parabola $y=-x^2+4x-3$.
（The coordinates of the vertex of paeabola ① are（$-a+2$, $-a^2+1$）. Since $x=-a+2$, $y=-a^2+1$, by eliminating a , we can find $y=-x^2+4x-3$.）

① When the value of a is 3.6



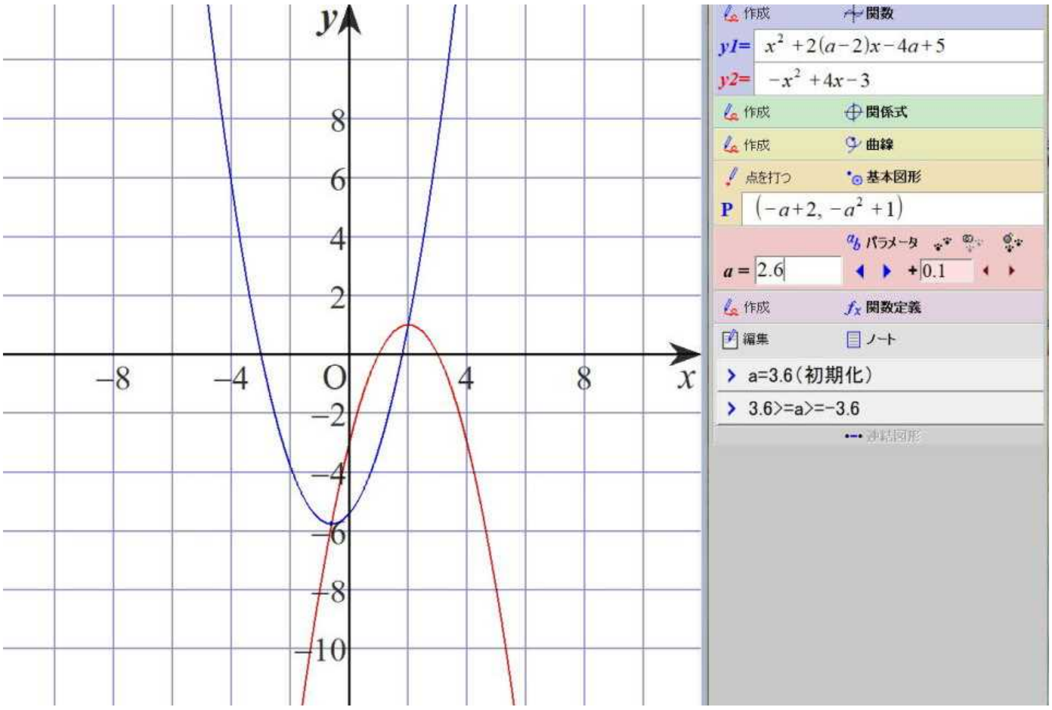
Interesting Simulation Ⅰ（Grapes）

2024.2.2
Sohun

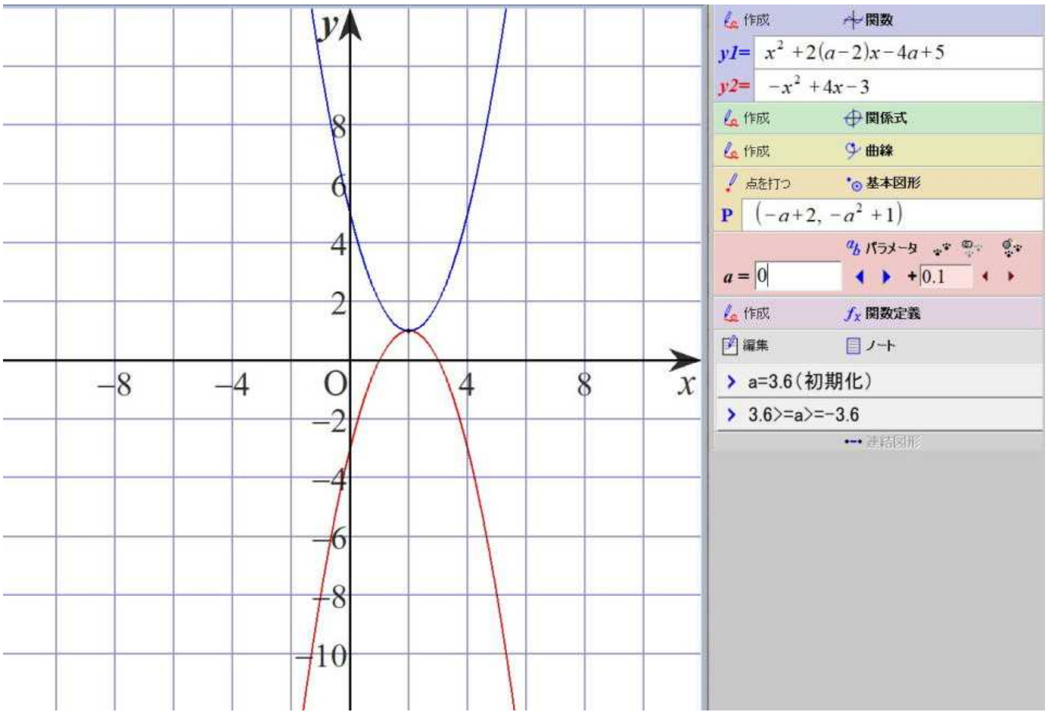
8　Locus of the vertex of a parabola

（２）Experimental result（Grapes version simulation）

② When the value of a is 2.6



③ When the value of a is 0



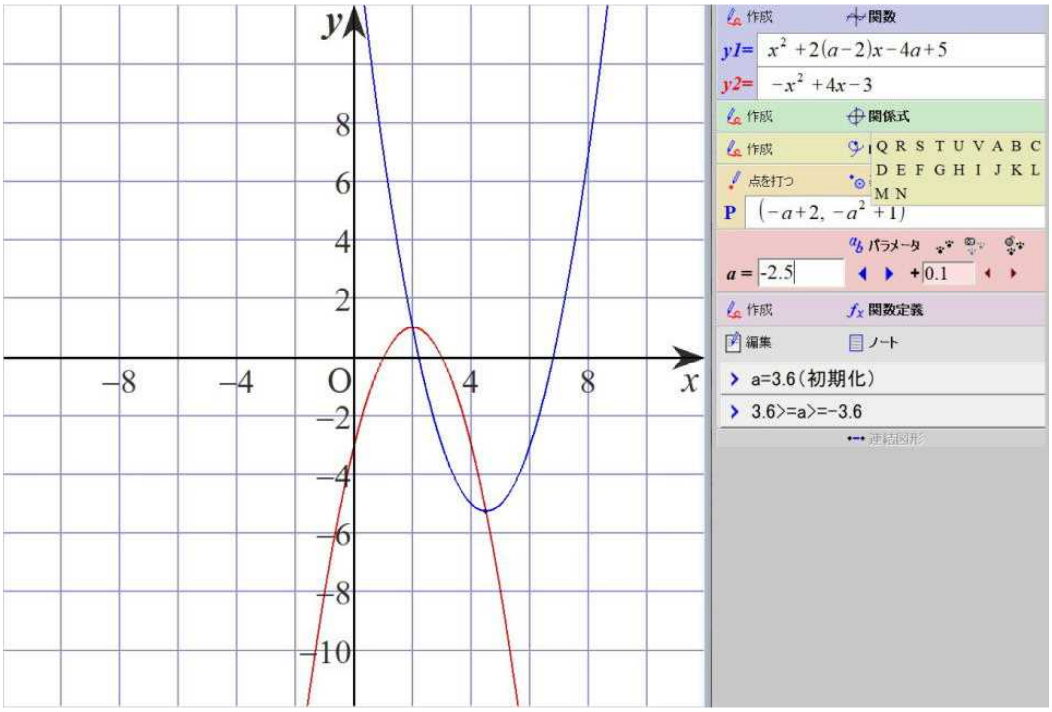
Interesting Simulation Ⅰ（Grapes）

2024.2.2
Sohun

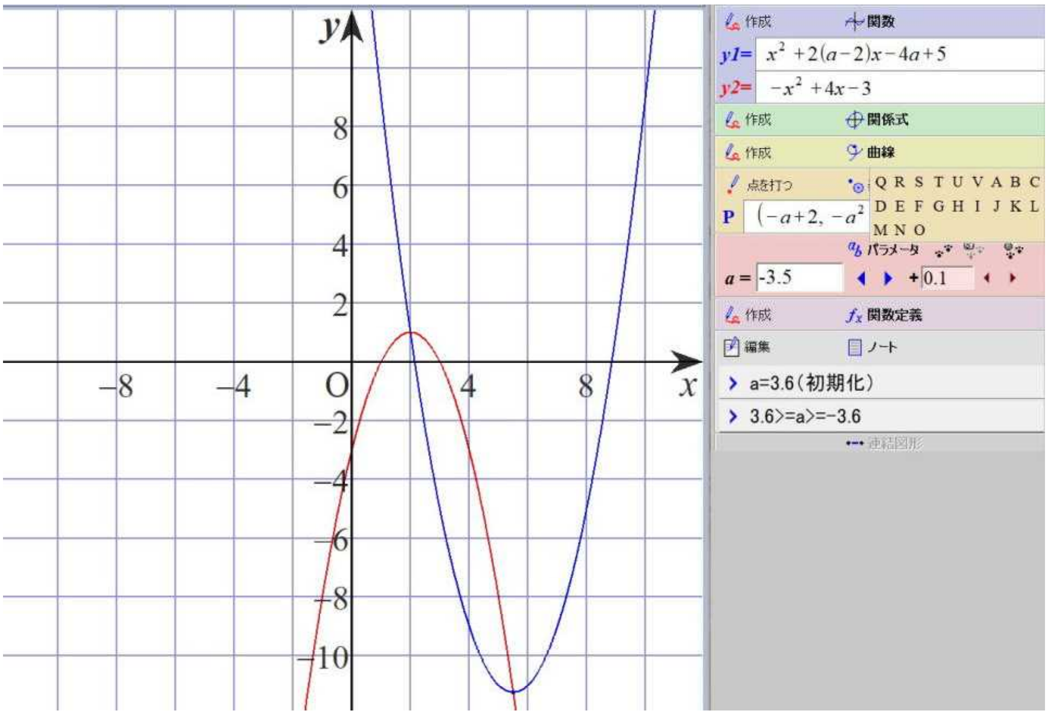
８　Locus of the vertex of a parabola

（２）Experimental result（Grapes version simulation）

④ When the value of a is -2.5



⑤ When the value of a is -3.5



Interesting Simulation Ⅰ (Grapes)

2024.2.3
Sohun

9 Number of common points between straight line and circle

(1) Exam question 9

Straight line: y=mx+1 ・・・①

Circle: x ²+y ²-2x+2y+1=0 ・・・②

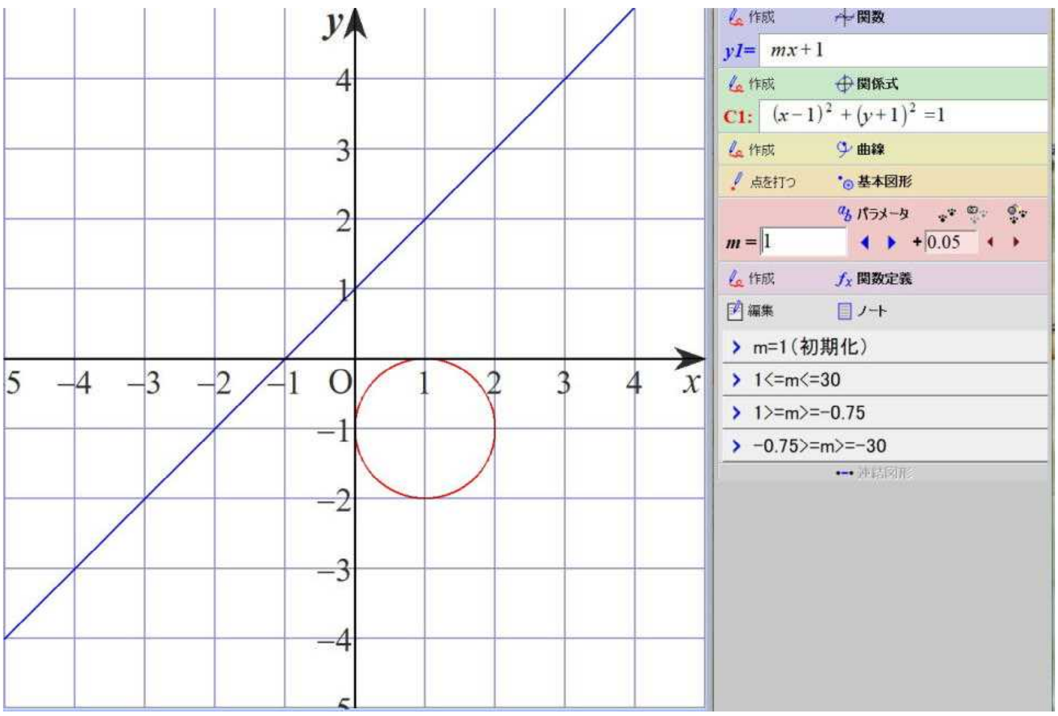
Find the number of the common points between straight line ① and circle ②.

(2) Experimental result (**Grapes** version simulation)

【Experiment day】
February 3 , 2024
【PC used】
Lavie NX850 / N
【GRAPES used】
GRAPES 7.84
【Script used】
Self-made file
『examquestion9.gps』

【Consideration】
I varied the value of m from 30 to -30 and observed common points between straight line ① and circle ②. When m>-0.75 , straight line ① and circle ② are far apart. When m = -0.75 , straight line ① touches circle ② at one point. When m<-0.75 , straight line ① intersects circle ② at two points.
(The value of m when straight line ① touches circle ② can be found from the fact that the distance between the center of circle ② and straight line ① is equal to the radius of circle ②.
m = -3/4 (-0.75))
Therefore
When m > -3/4 , 0
When m = -3/4 , 1
When m < -3/4 , 2

① When the value of m is 1



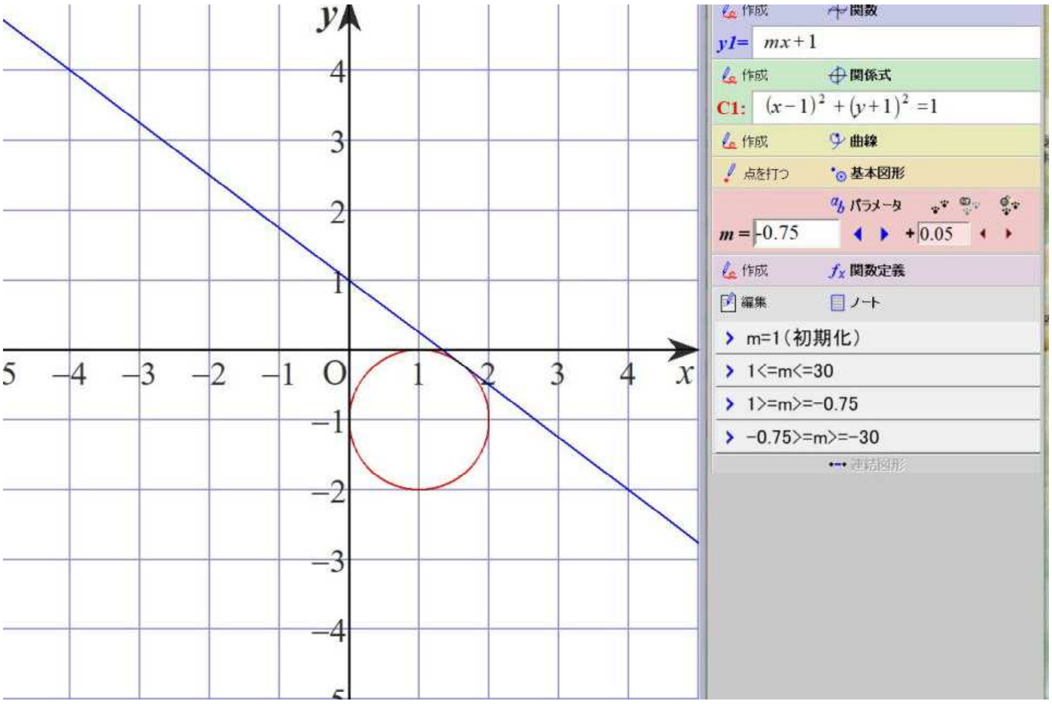
Interesting Simulation Ⅰ（Grapes）

2024.2.3
Sohun

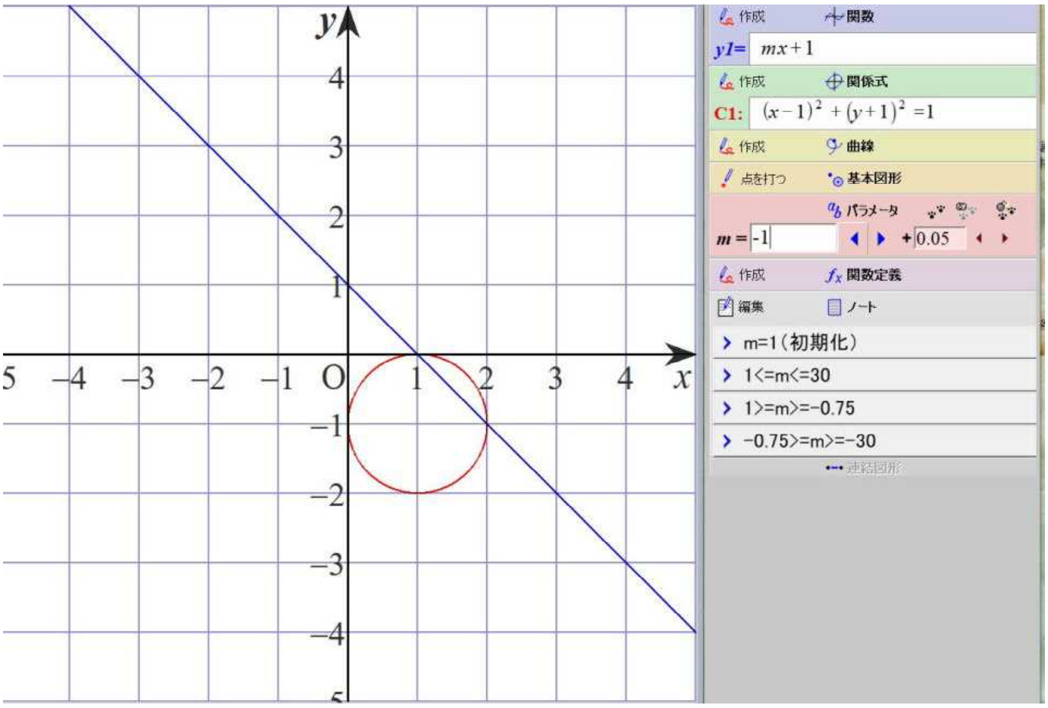
９　Number of common points between straight line and circle

（２）Experimental result（Grapes version simulation）

② When the value of m is -0.75



③ When the value of m is -1



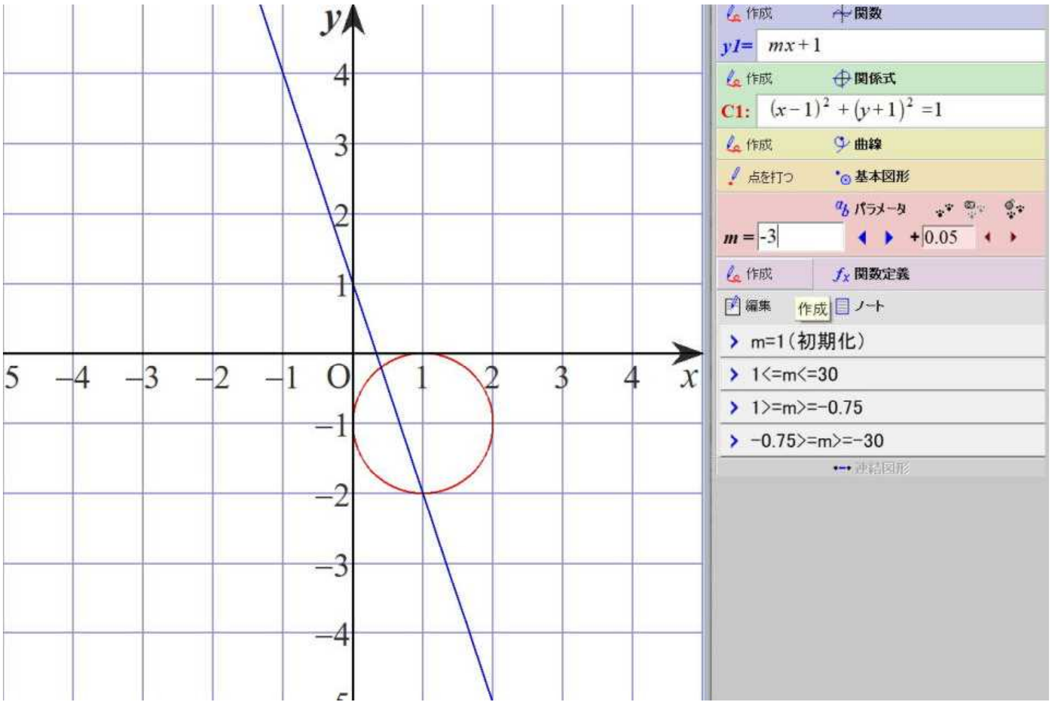
Interesting Simulation Ⅰ（Grapes）

2024.2.3
Sohun

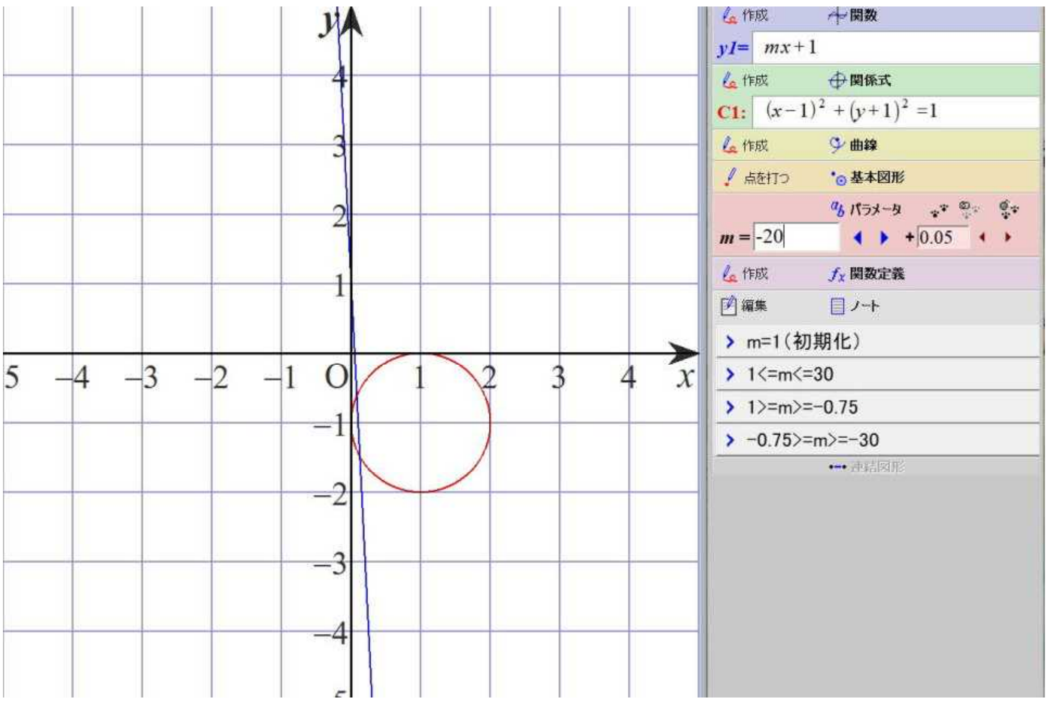
９　Number of common points between straight line and circle

（２）Experimental result（Grapes version simulation）

④ When the value of m is -3



⑤ When the value of m is -20



Interesting Simulation Ⅰ (Grapes)

2024.2.4
Sohun

１０ Locus of intersection of two straight lines

（１） Exam question 10

When t changes as a real number , what kind of figure will the intersection point P(x,y) of two straight lines L: tx-y=t and M: x+ty=2t+1 have ?

Find the equation and illustrate it.

（２） Experimental result （Grapes version simulation）

【Experiment day】
February 4 , 2024
【PC used】
Lavie NX850 / N
【GRAPES used】
GRAPES 7.84
【Script used】
Self-made file
『examquestion10.gps』

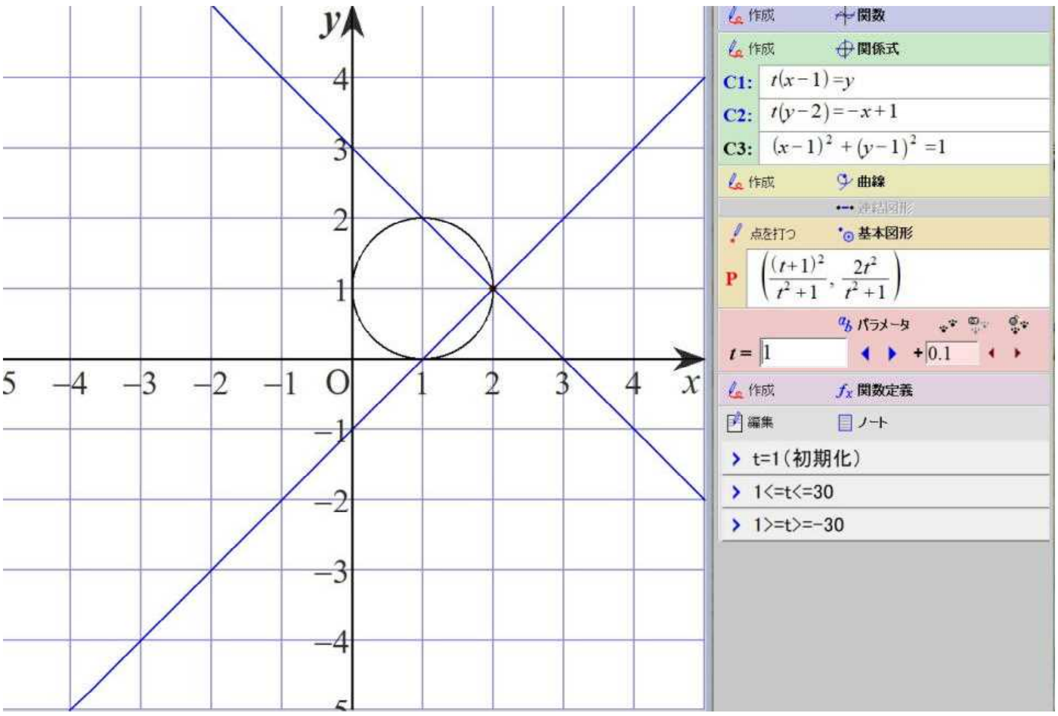
【Consideration】

I varied the value of t from -30 to 30 and observed the intersection point P of straight lines L and M. When t>1 , the intersection point P of two straight lines L and M is on the counterclockwise arc of the circle ①: $(x-1)^2+(y-1)^2=1$ whose ends are points (2,1) and (1,2) on the circle ① (However , both ends are excluded). When t<1 , the intersection point P of the two straight lines L and M is on the clockwise arc of the circle ① whose ends are points (2,1) and (1,2) on the circle ① (However , both ends are excluded). When t=1 , the coordinates of the intersection P of the two straight lines L and M are (2,1).

Therefore , the found figure is a circle $(x-1)^2+(y-1)^2=1$ (excluding point(1,2)).

(Solve for x and y by combining tx-y=tx+t , y=2t+1. $x=(t^2+2t+1)/(t^2+1)$、$y=2t^2/(t^2+1)$ and the equation for circle ① can be found.)

① When the value of t is 1



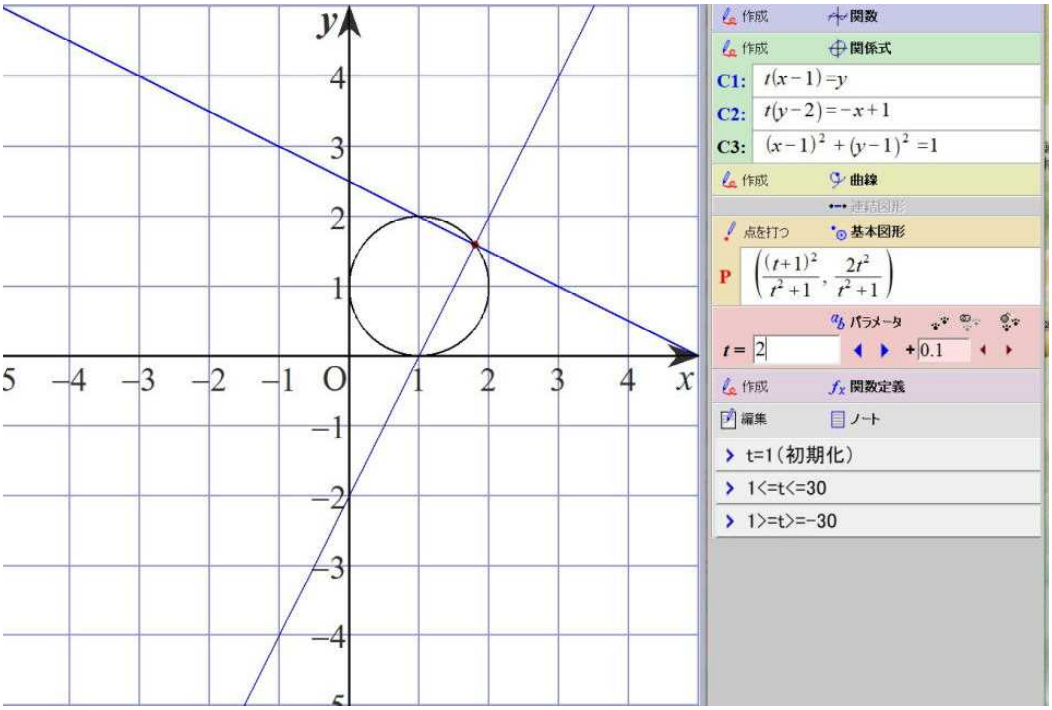
Interesting Simulation Ⅰ（Grapes）

2024.2.4
Sohun

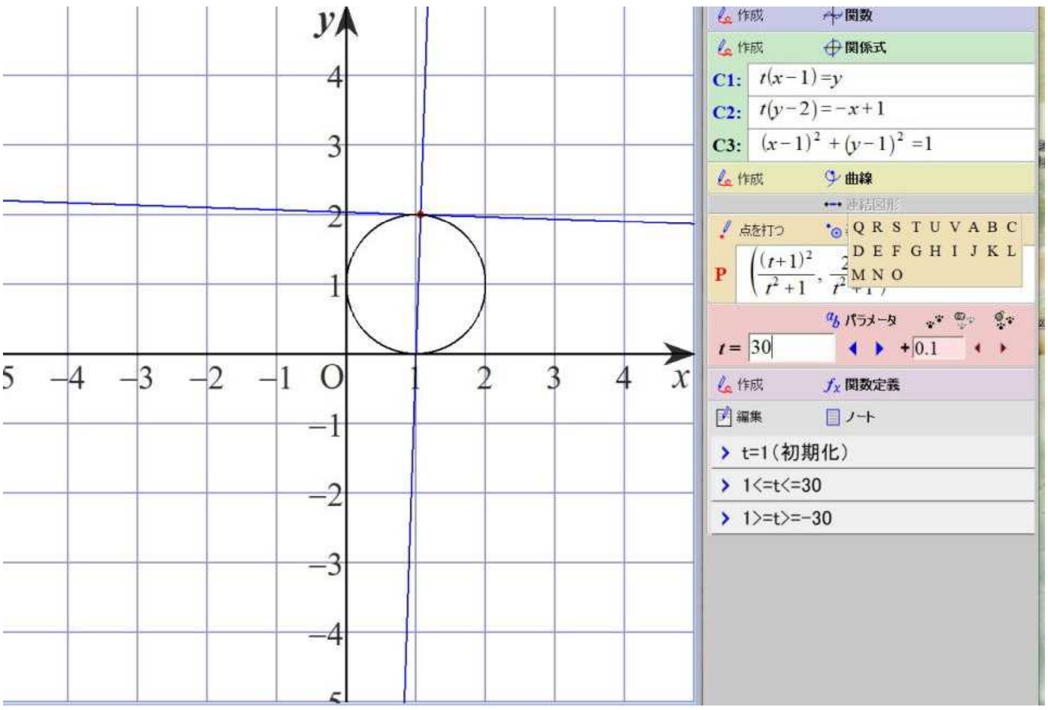
１０　Locus of intersection of two straight lines

（２）Experimental result（Grapes version simulation）

② When the value of t is 2



③ When the value of t is 30



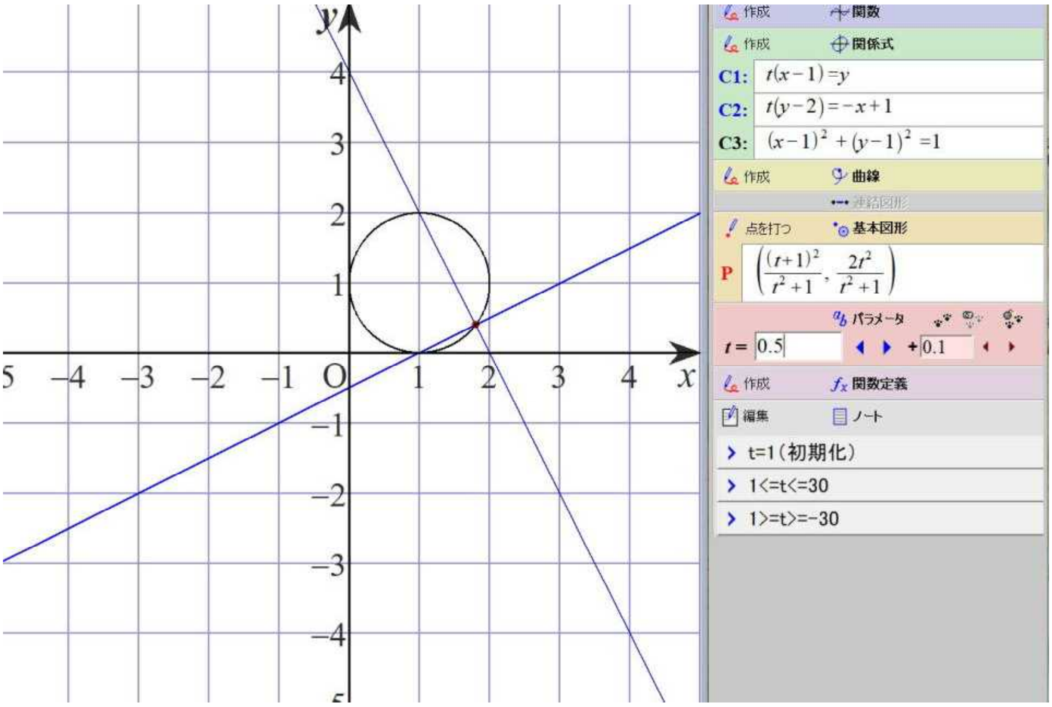
Interesting Simulation Ⅰ（Grapes）

2024.2.4
Sohun

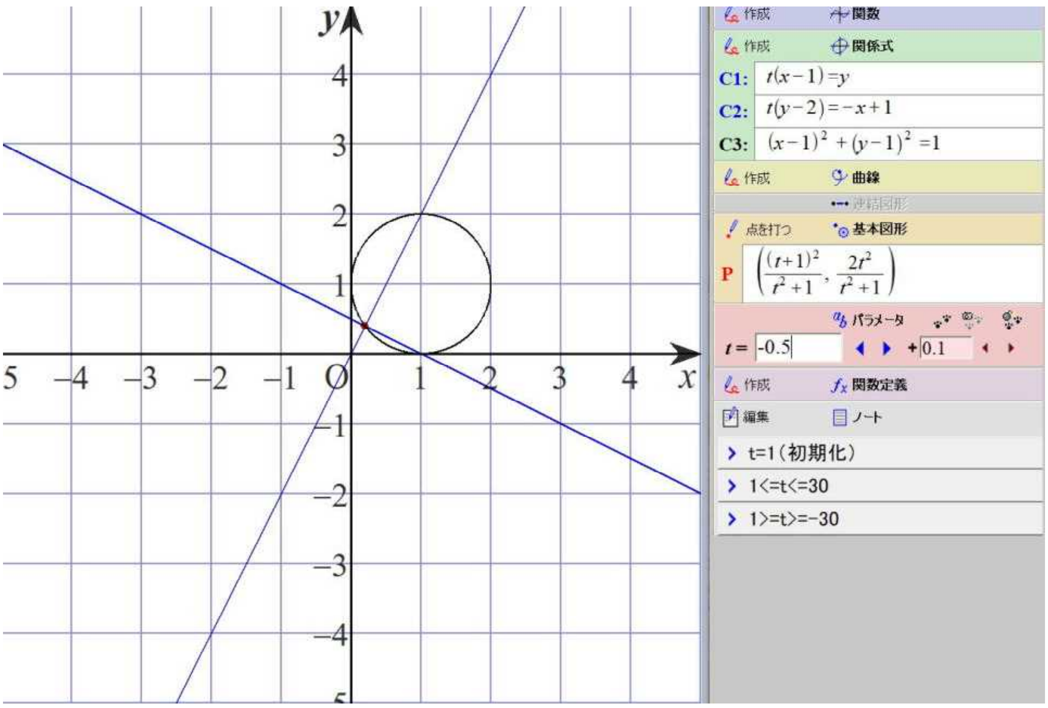
１０　Locus of intersection of two straight lines

（２）Experimental result（Grapes version simulation）

④ When the value of t is 0.5



⑤ When the value of t is -0.5



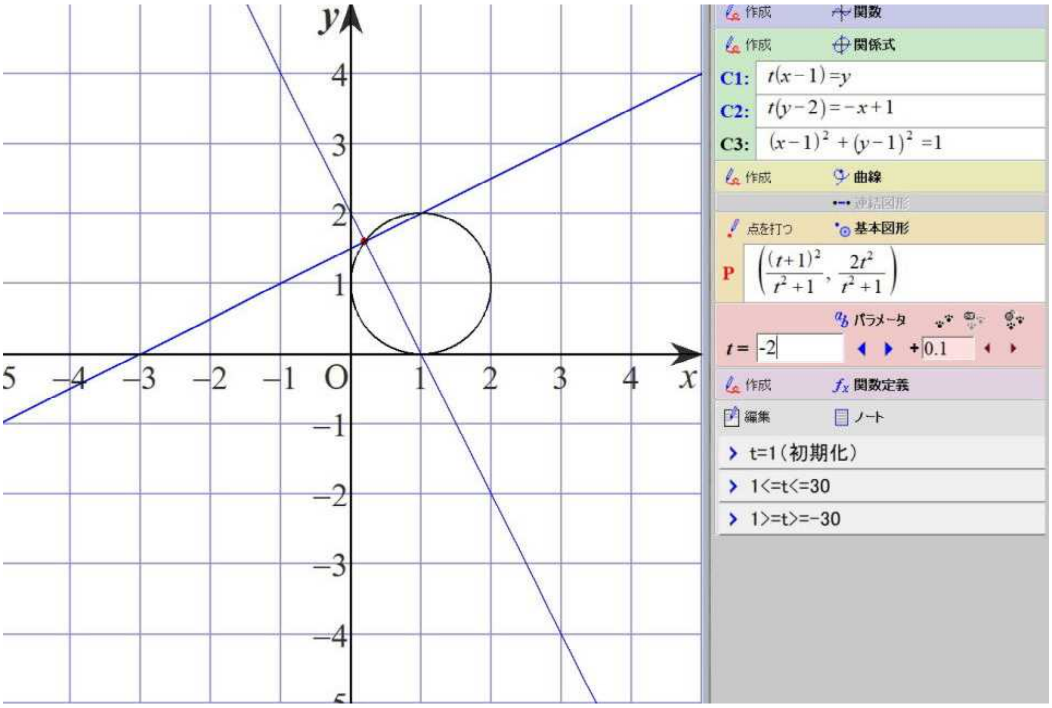
Interesting Simulation Ⅰ（Grapes）

2024.2.4
Sohun

１０　Locus of intersection of two straight lines

（２） Experimental result（Grapes version simulation）

⑥ When the value of t is -2



⑦ When the value of t is -30