

Interesting Simulation II (Grapes)

2.7.2024
Sohun

1 1 Locus 2 of intersection of two straight lines

(1) Exam question 11

When t changes while taking the value of a real number, regarding the two straight lines
 $L : x+t(y-3)=0$, $M : tx-(y+3)=0$

- ① Show that the straight line L passes through a fixed point regardless of the value of t .
- ② When t moves through the real numbers, what shape does the intersection of straight lines L and M draw?

(2) Experimental result (Grapes version simulation)

【Experiment day】

February 7, 2024

【PC used】

Lavie NX850/N

【GRAPES used】

GRAPES 7.84

【Script used】

Self-made file

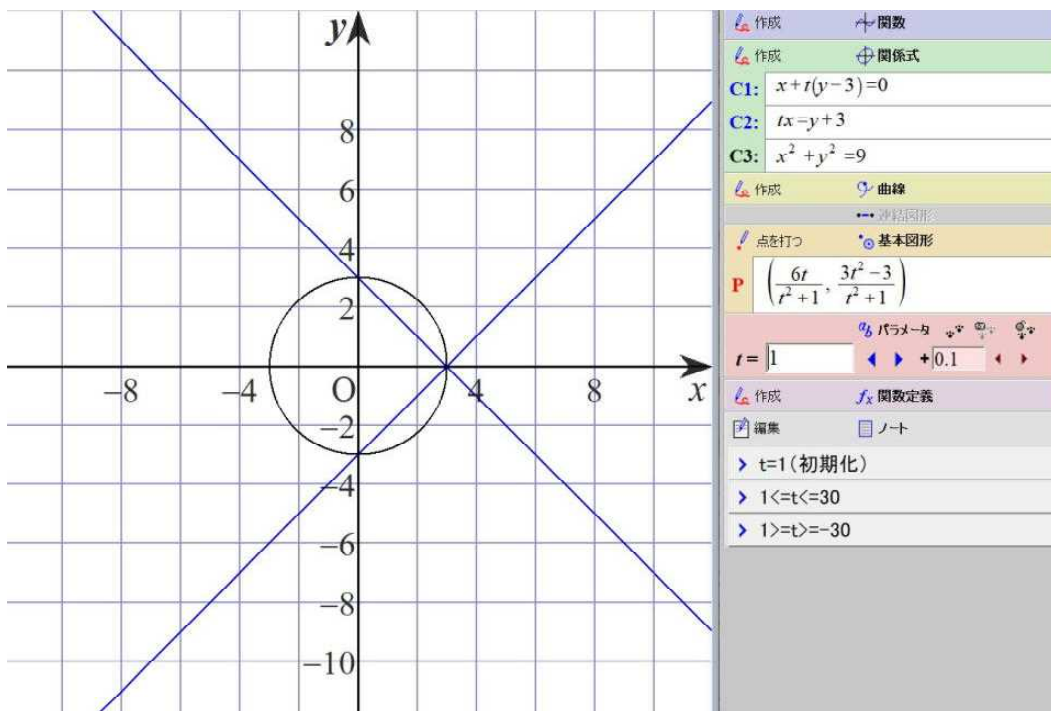
『examquestion11.gps』

【Consideration】

I varied the value of t from -30 to 30 and observed the intersection P of two straight lines L and M . When $t > 1$, the intersection P of the two straight lines L and M on the counterclockwise arc of the circle ①: $x^2 + y^2 = 9$ whose ends are points $(3,0)$ and $(0,3)$ on the circle ①.

(However, both ends are excluded.) When $t < 1$, the intersection P of the two straight lines L and M on the counterclockwise arc of the circle ① whose ends are points $(3,0)$ and $(0,3)$ on the circle ①. (However, both ends are excluded.) When $t = 1$, the coordinates of the intersection P of the two straight lines L and M are $(3,0)$. Therefore, the founded shape is a circle: $x^2 + y^2 = 9$ (point $(3,0)$ is excluded). Combine $x+t(y-3)=0$ and $tx-(y+3)=0$, and solve for x and y .
 $x = 6t / (t^2 + 1)$, $y = 3(t^2 - 1) / (t^2 + 1)$, and equation for circle ① can be derived.)

① When the value of t is 1



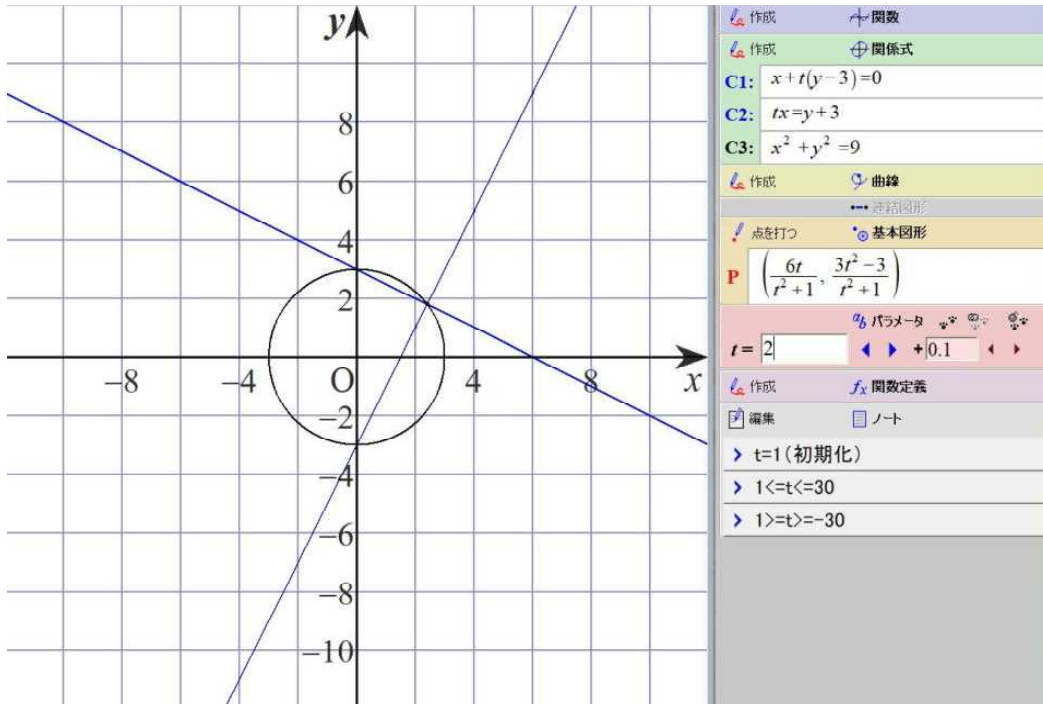
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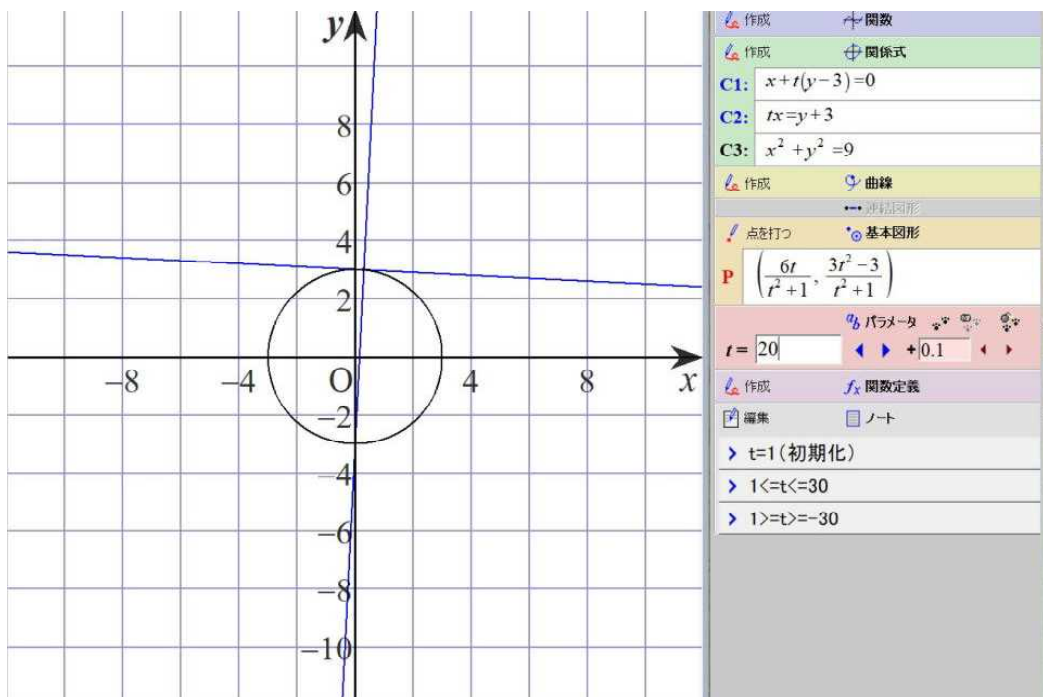
1 1 Locus 2 of intersection of two straight lines

(2) Experimental result (Grapes version simulation)

② When the value of t is 2



③ When the value of t is 20



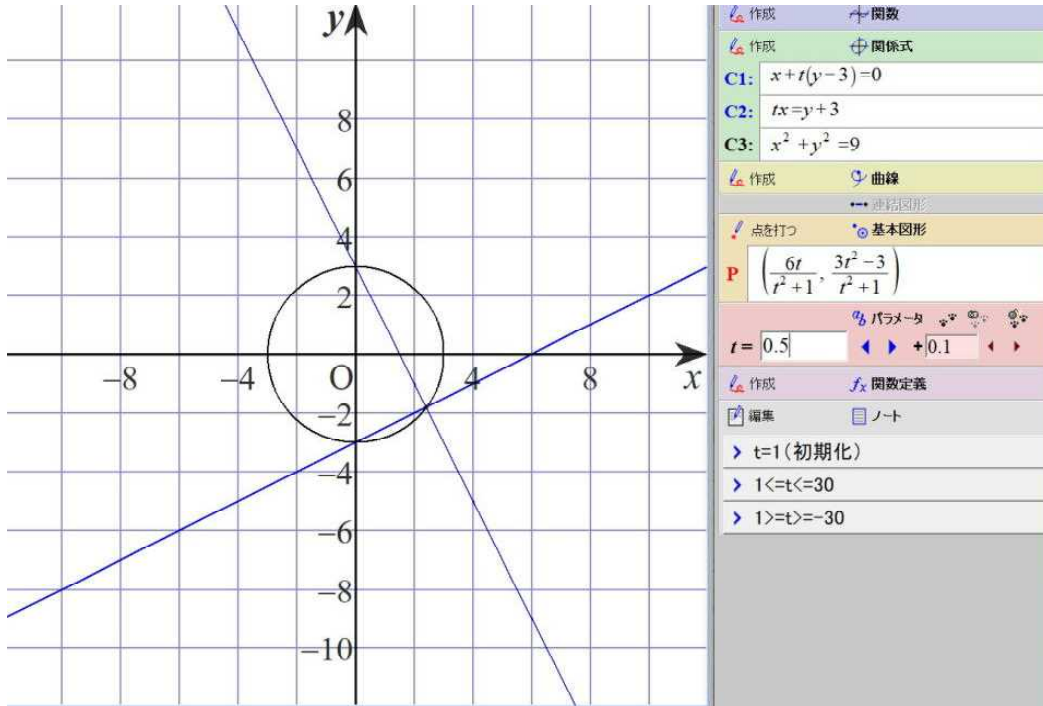
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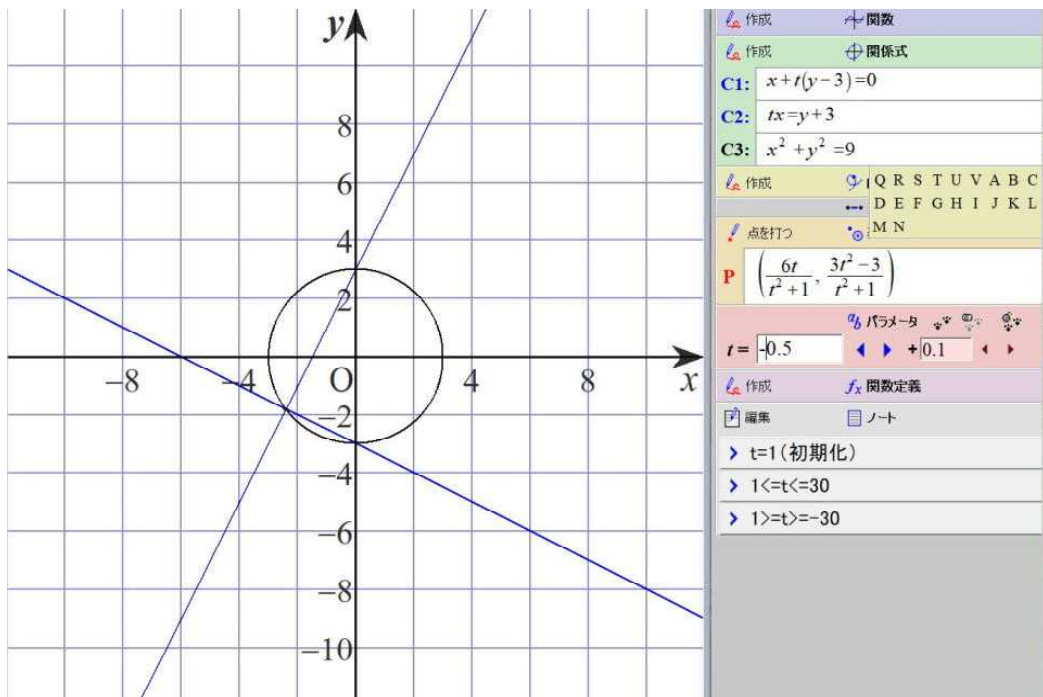
1 1 Locus 2 of intersection of two straight lines

(2) Experimental result (Grapes version simulation)

④ When the value of t is 0.5



⑤ When the value of t is -0.5



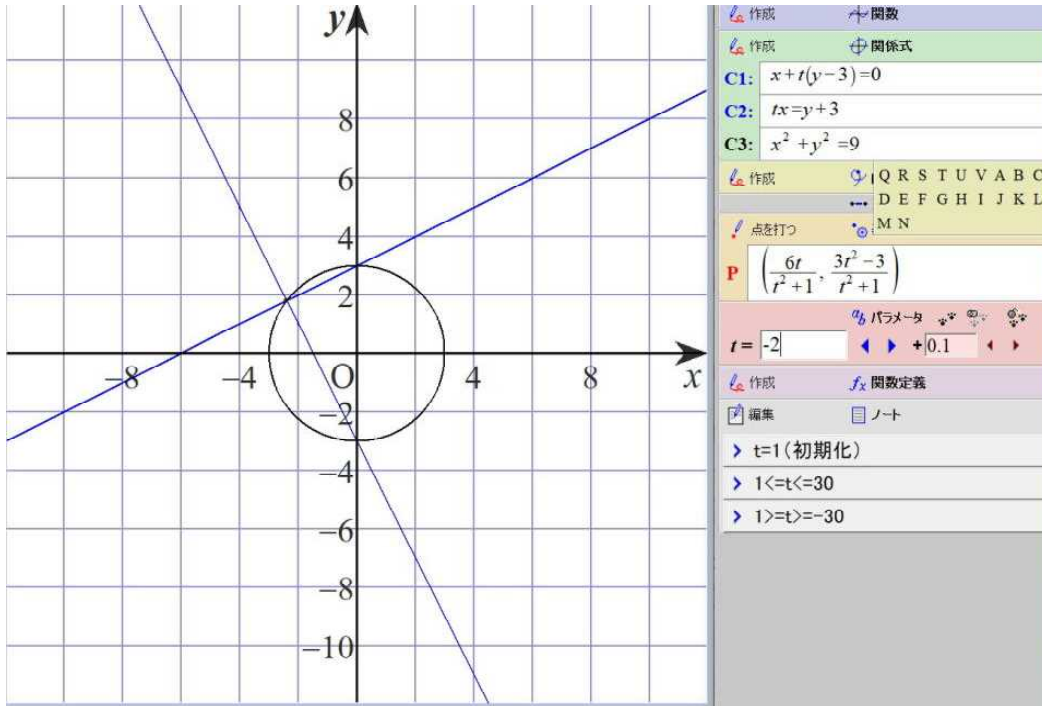
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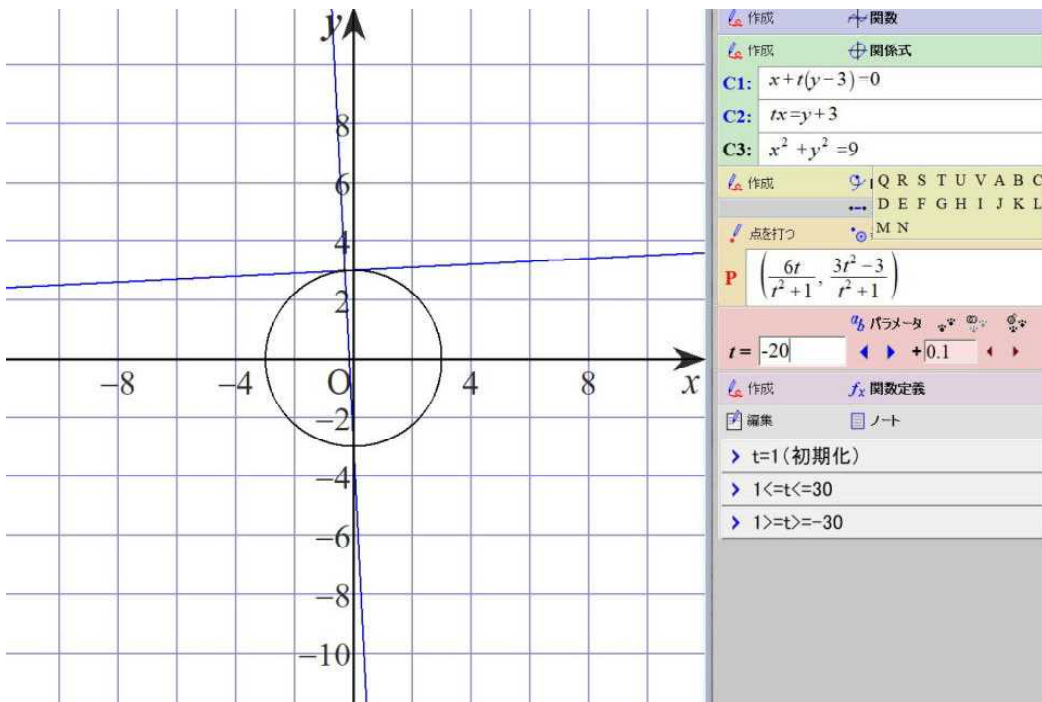
1 1 Locus 2 of intersection of two straight lines

(2) Experimental result (Grapes version simulation)

⑥ When the value of t is -2



⑦ When the value of t is -20



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1 2 Locus of the midpoint of the intersections of a straight line and a parabola

(1) Exam question 12

Suppose that the straight line $y=mx$ intersects the parabola $y=x^2+1$ at two different points P and Q.

- ① When m changes while satisfying this condition, find the range of possible values of a .
- ② At this time, find the locus of the midpoint M of the line segment PQ.

(2) Experimental result (Grapes version simulation)

【Experiment day】

February 8, 2024

【PC used】

Lavie NX850/N

【GRAPES used】

GRAPES 7.84

【Script used】

Self-made file

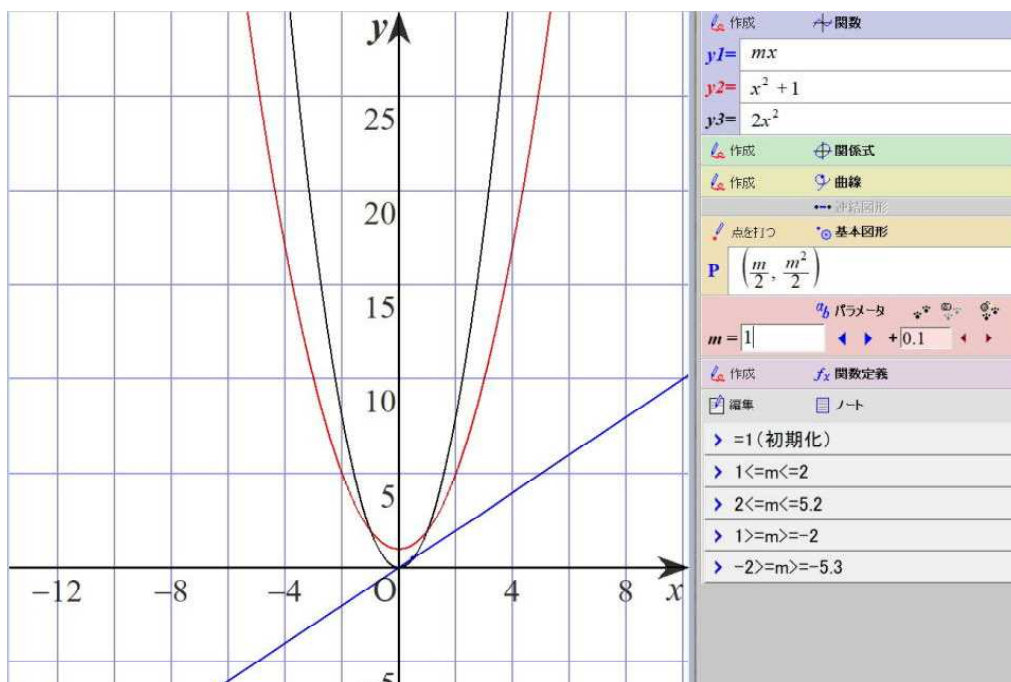
『examquestion12.gps』

【Consideration】

I varied the value of m from -5.3 to 5.2 and observed the common points of the straight line $y=mx$ and the parabola $y=x^2+1$, and the locus of the midpoint M between the intersections P and Q. When $m=2$ and $m=-2$, the straight line touches the parabola at one point. When $-1 < m < 1$, there is no common point between the straight line and the parabola. When $m < -2$, $2 < m$, the straight line intersects the parabola at two different points. (By combining $y=mx$ and $y=x^2+1$, the values of $m=2$ and $m=-2$ when they touch can be found from the multiple solution condition. In the same way, based on the condition that there are two different real solutions, we can find the range of m values $m < -2$, $2 < m$ when they intersect at two different points. From the coordinates $x=m/2$, $y=m^2/2$ of the midpoint M of PQ, $y=2x^2$ can be found.)

Therefore, the locus of the midpoint M is a parabola $y=x^2$ ($x < -1$, $1 < x$).

① When the value of m is 1



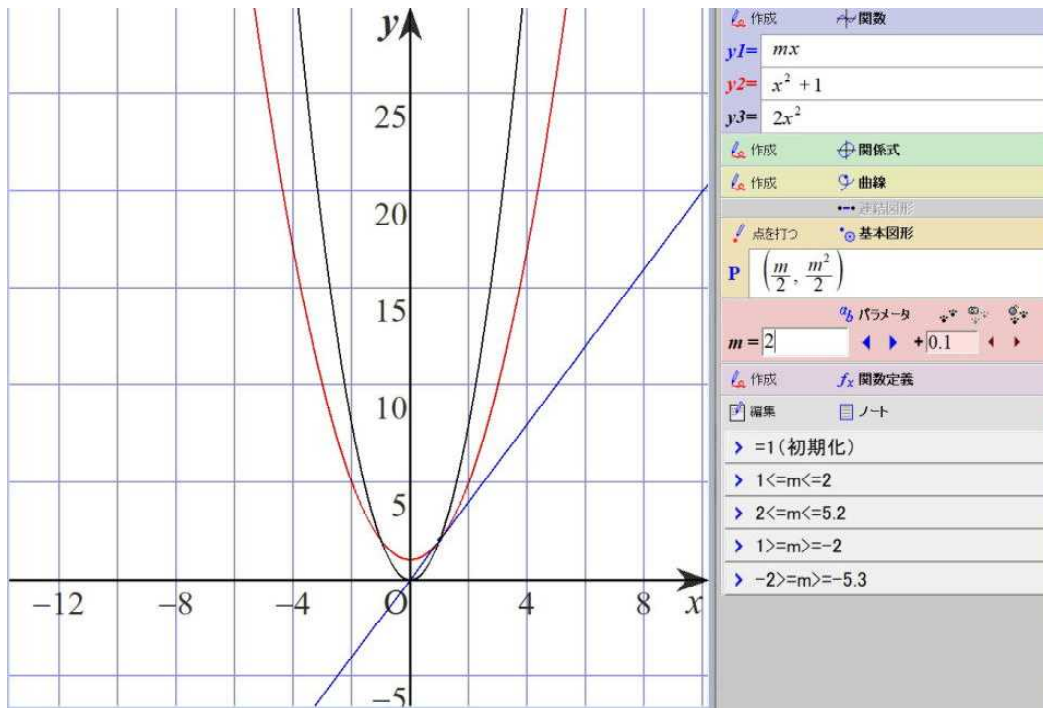
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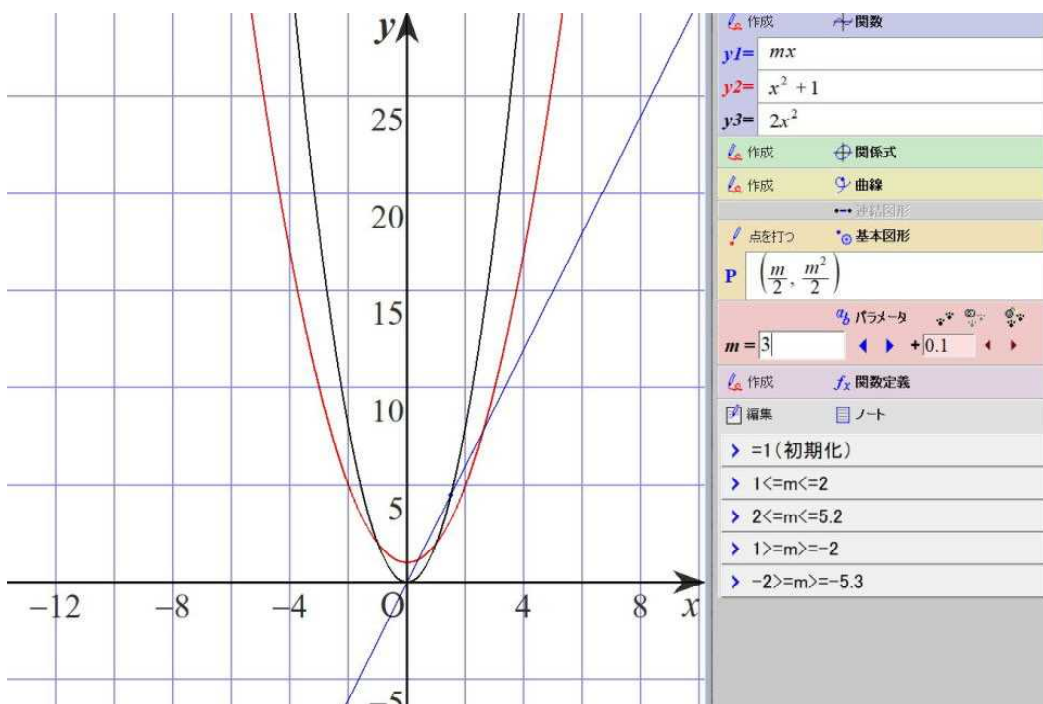
1 2 Locus of the midpoint of the intersections of a straight line and a parabola

(2) Experimental result (Grapes version simulation)

② When the value of m is 2



③ When the value of m is 3



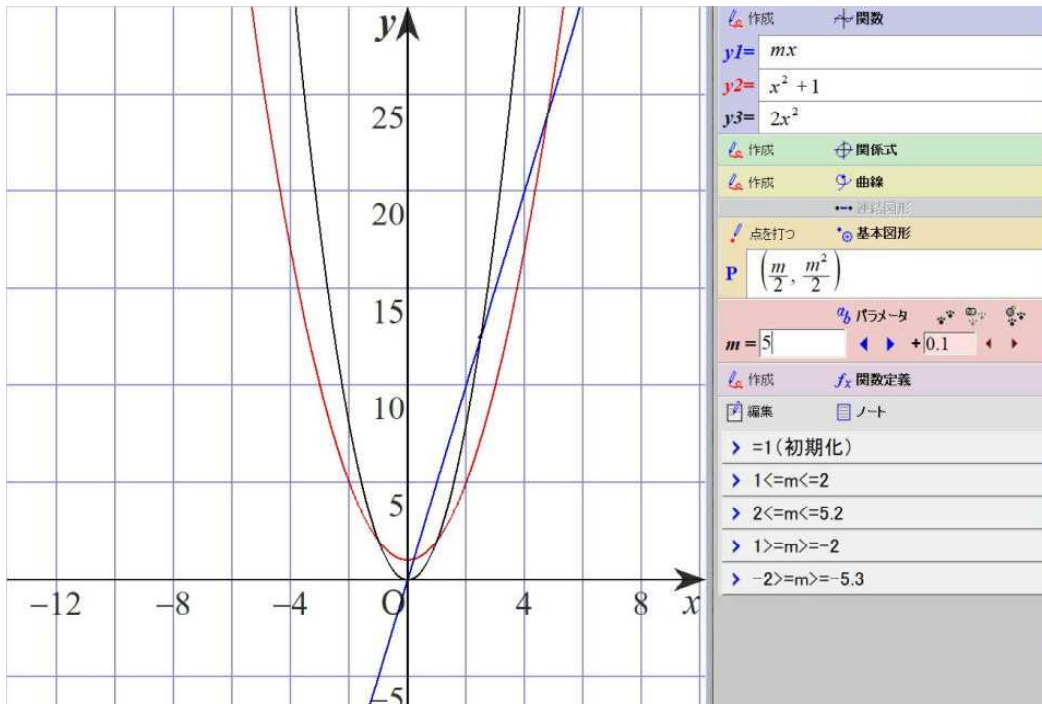
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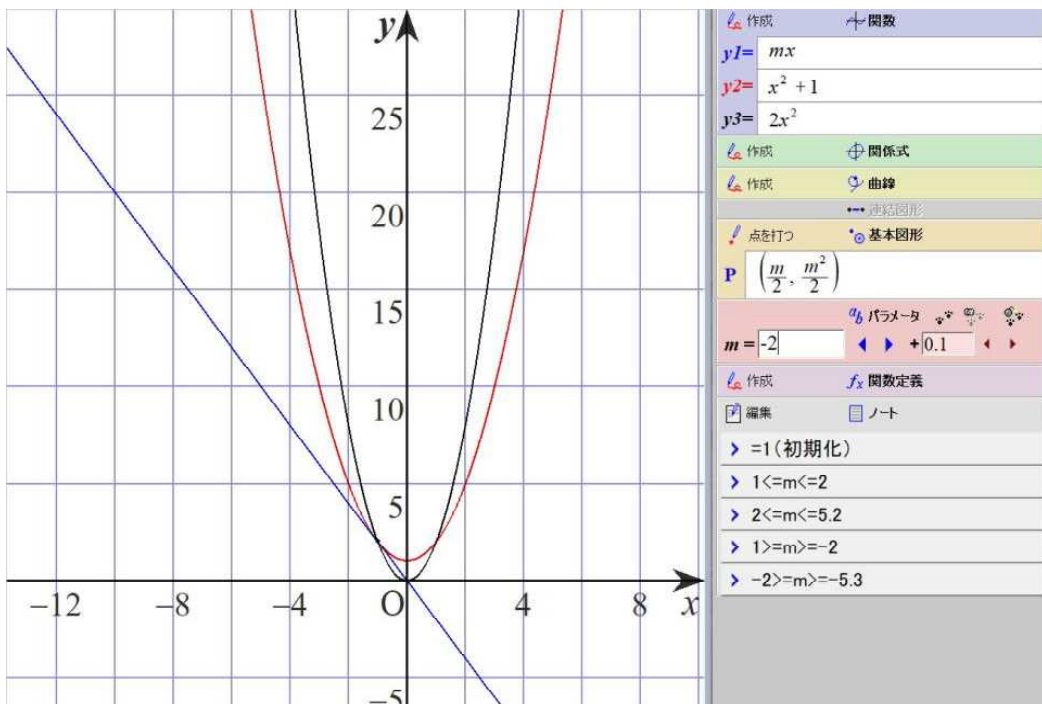
1 2 Locus of the midpoint of the intersections of a straight line and a parabola

(2) Experimental result (Grapes version simulation)

④ When the value of m is 5



⑤ When the value of m is -2



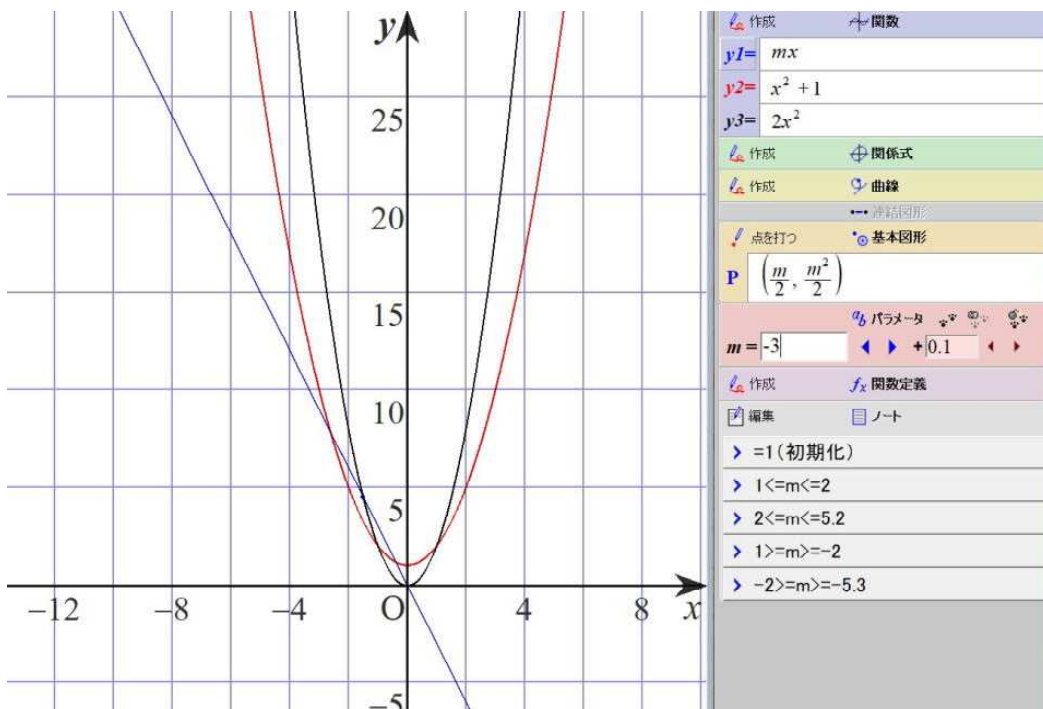
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2.8.2024
Sohun

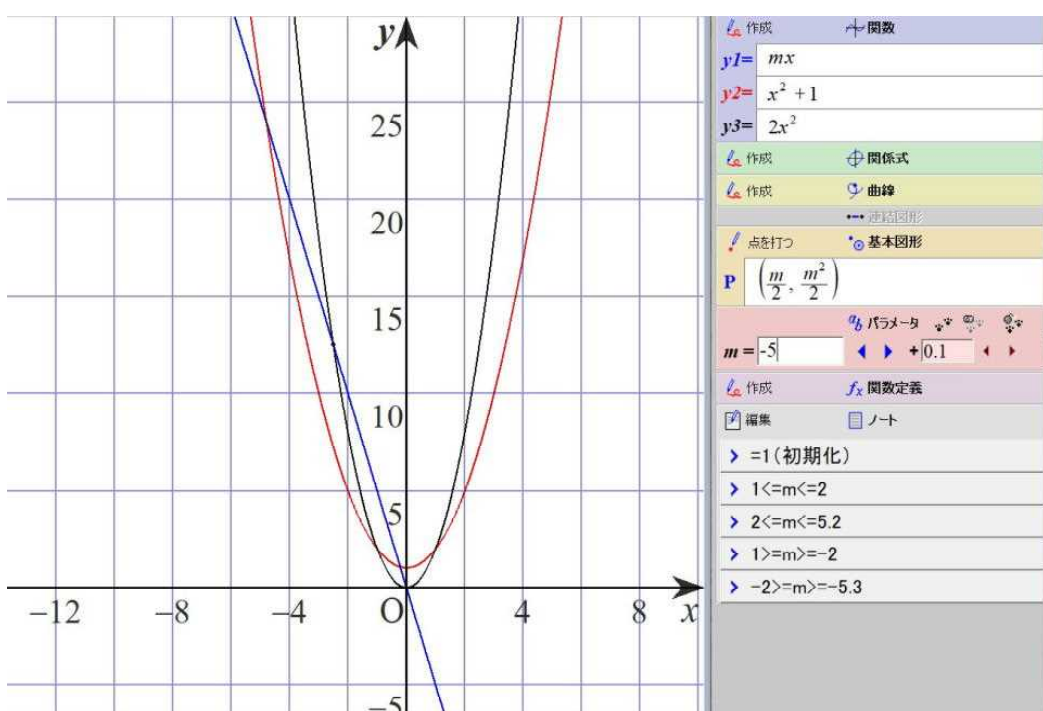
1 2 Locus of the midpoint of the intersections of a straight line and a parabola

(2) Experimental result (Grapes version simulation)

⑥ When the value of m is -3



⑦ When the value of m is -5



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1.3 Parallel and perpendicular conditions

(1) Exam question 13

Find the value of the constant k when the two lines ①: $2x+5y-3=0$, ②: $5x+ky-2=0$ are parallel and perpendicular, respectively.

(2) Experimental result (Grapes version simulation)

【Experiment day】

February 9, 2024

【PC used】

Lavie NX850/N

【GRAPES used】

GRAPES 7.84

【Script used】

Self-made file

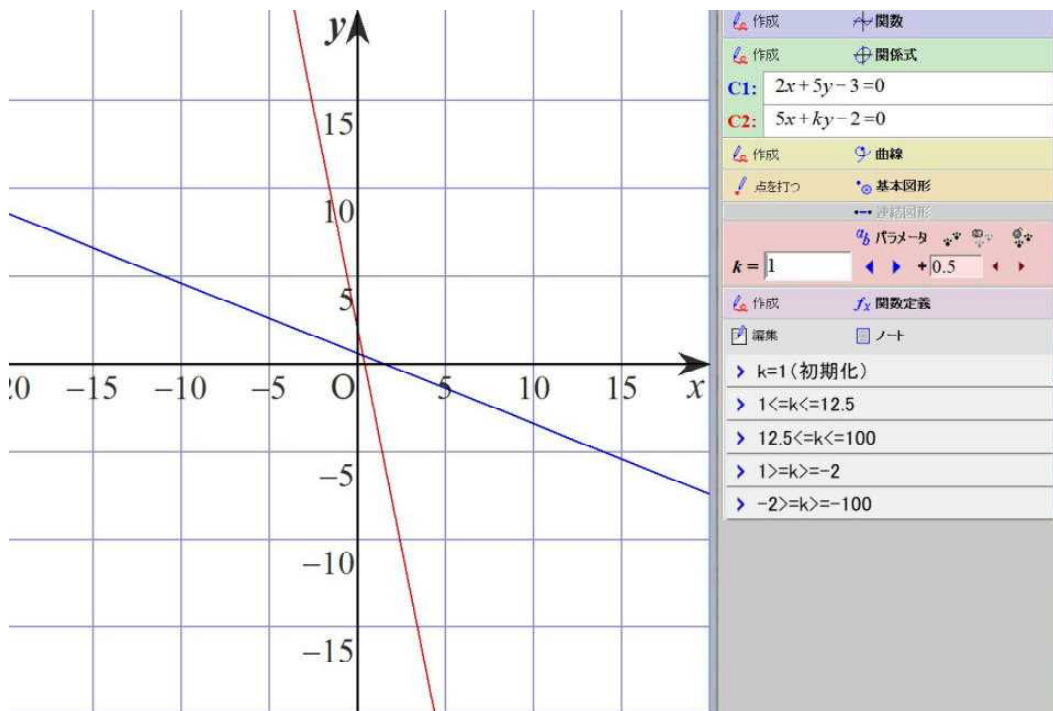
『examquestion13.gps』

【Consideration】

I varied the value of k from -100 to 100 and observed the positional relationship between straight line ① and straight line ②. When $k=12.5$, straight line ① and straight line ② are parallel. When $k=-2$, straight line ① and straight line ② are perpendicular. Also, when the value of k is other than 12.5 , the two straight lines are not parallel. Furthermore, the two straight lines are not perpendicular when the value of k is other than -2 .

(Generally, when two straight lines $ax+by+c=0$ and $dx+ey+f=0$ are parallel, $ae=bd$ holds true. When perpendicular, $ad+be=0$ holds true. Therefore, from $2k=5 \times 5$, $k=12.5$ can be found when they are parallel. From $2 \times 5+5k=0$, $k=-2$ can be found when they are perpendicular.)

① When the value of k is 1



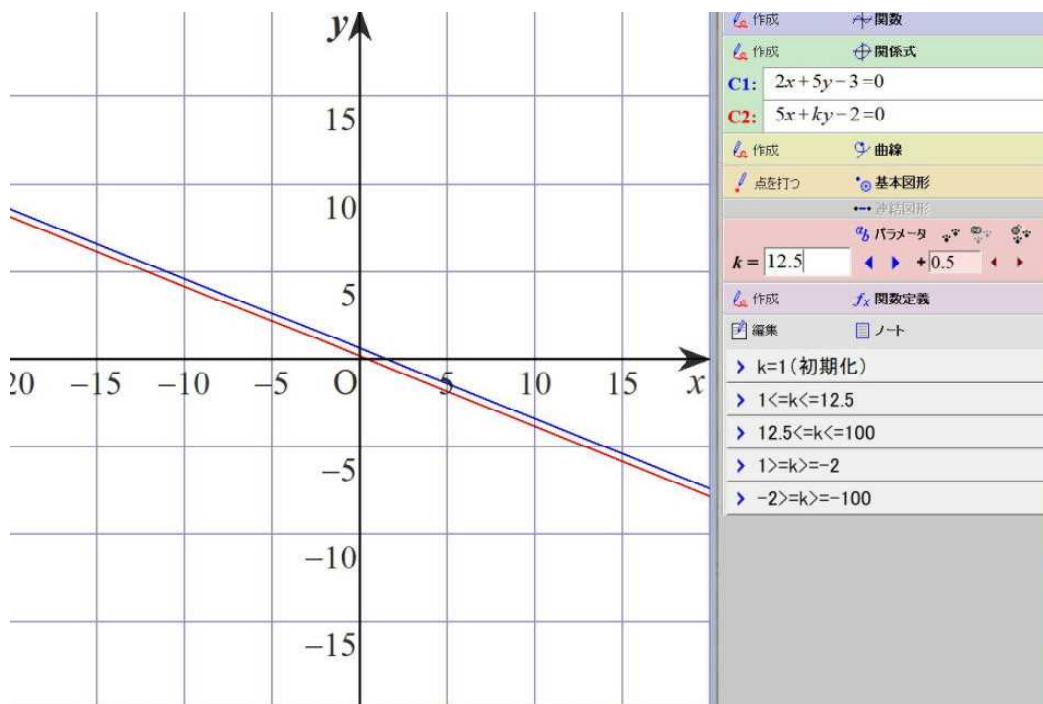
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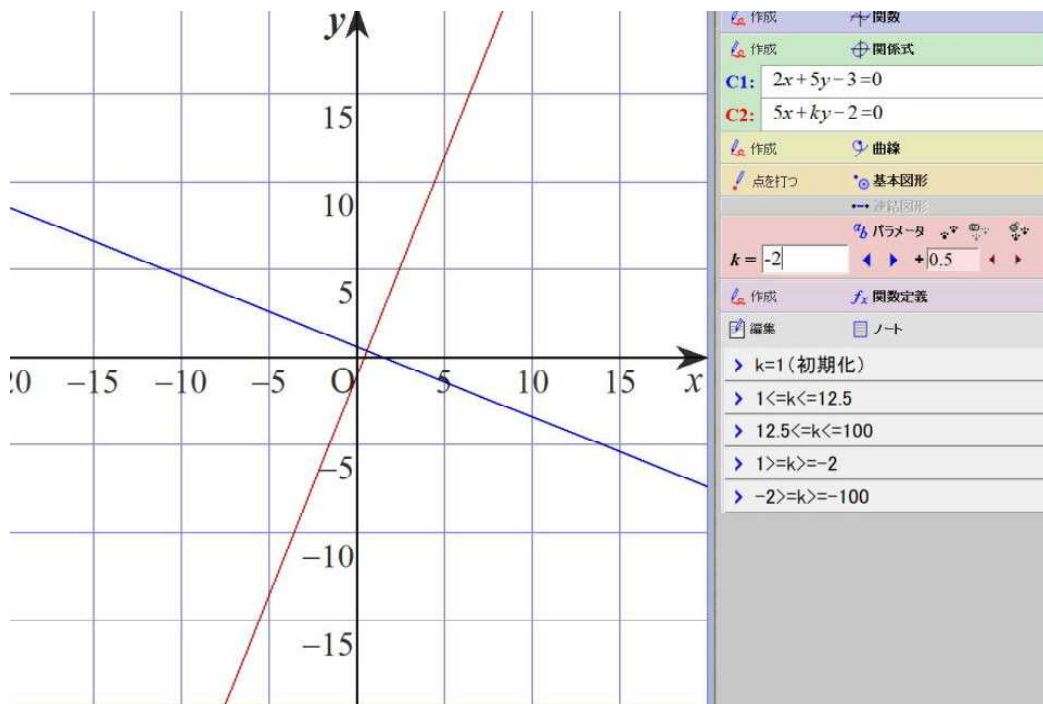
1.3 Parallel and perpendicular conditions

(2) Experimental result (Grapes version simulation)

② When the value of k is 12.5



③ When the value of k is -2



Interesting Simulation II (Grapes)

2.10.2024
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1 4 Straight line passing through 3 points

(1) Exam question 14

Find the value of the constant a when three points $A(a,-2)$, $B(3,2)$, $C(-1,4)$ are on the same straight line.

(2) Experimental result (Grapes version simulation)

【Experiment day】

February 10, 2024

【PC used】

Lavie NX850/N

【GRAPES used】

GRAPES 7.84

【Script used】

Self-made file

『examquestion14.gps』

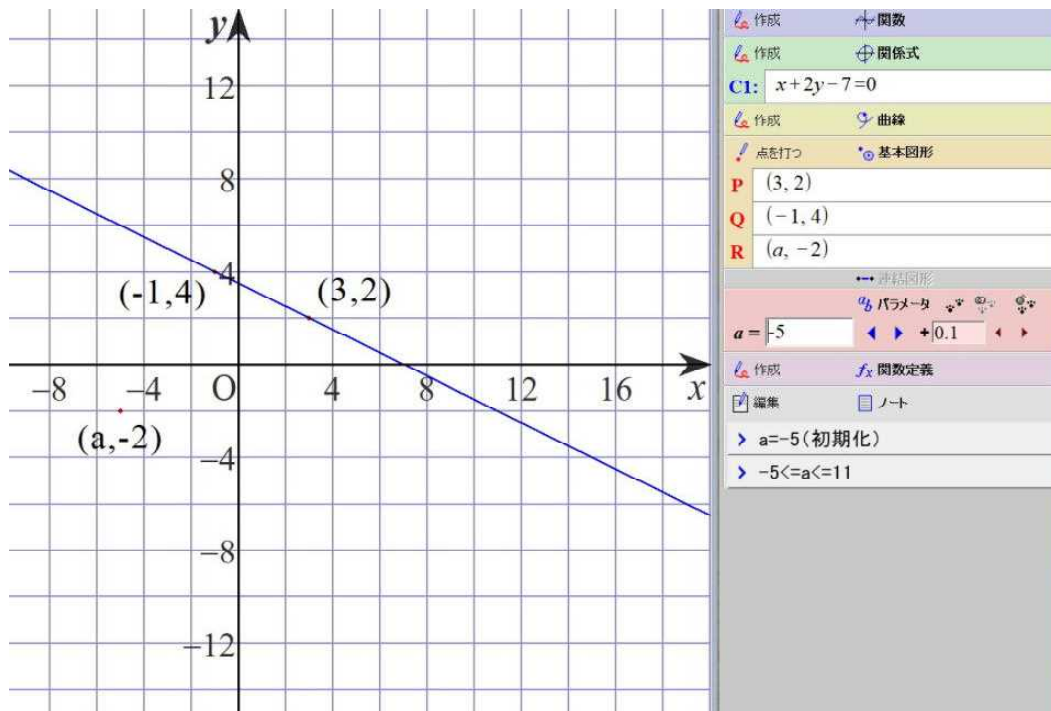
【Consideration】

I varied the value of a from -5 to 11 and observed the positional relationship between point $A(a,-2)$ and the straight line BC . When $a=11$, point $A(a,-2)$ is on the straight line BC .

Therefore, when $a=11$, three points A , B , and C are on the same straight line.

(Find the equation of the straight line passing through the two points B and C . Since the coordinates of point A $x=a$ and $y=-2$, satisfy this straight line equation, we can find $a=11$ by substitution.)

① When the value of a is -5



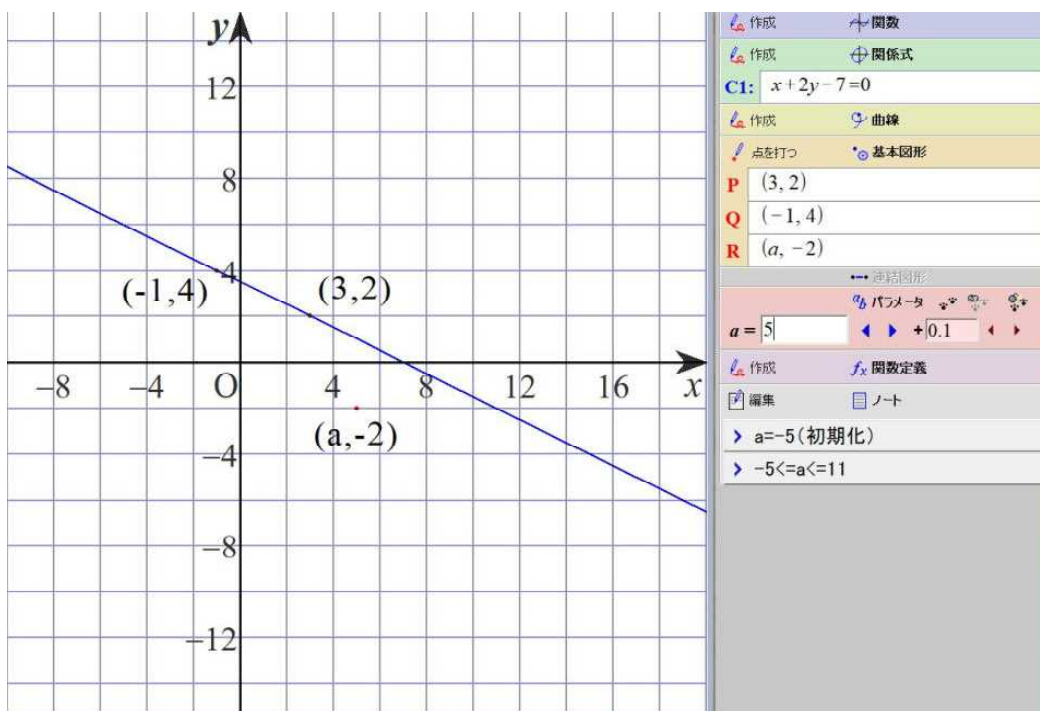
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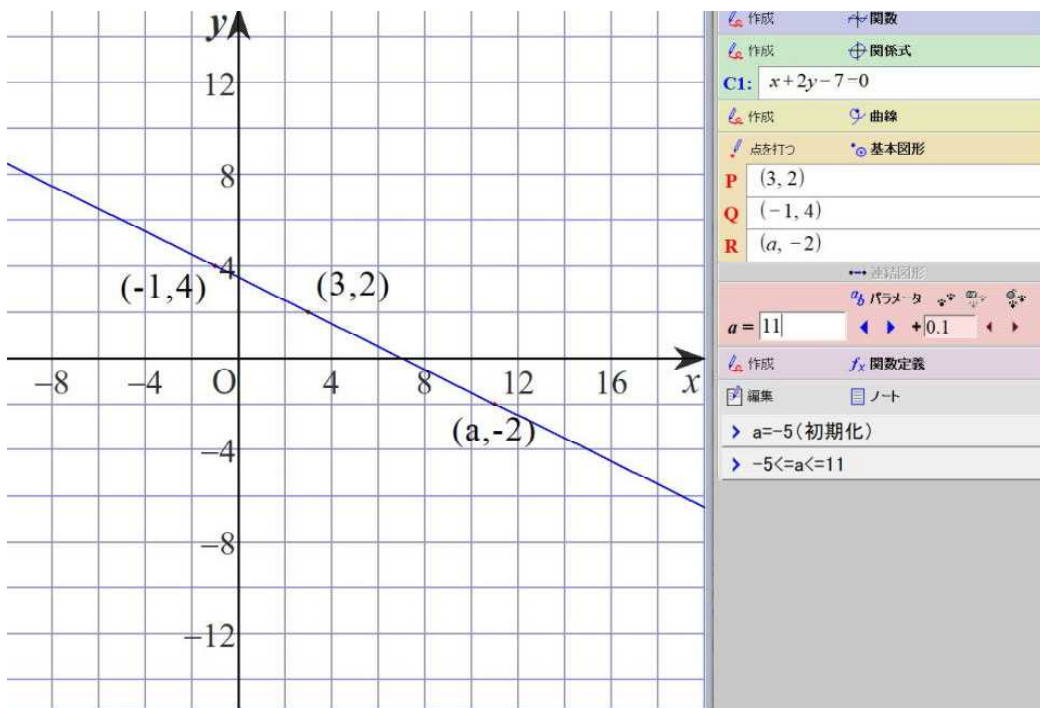
1 4 Straight line passing through 3 points

(2) Experimental result (Grapes version simulation)

② When the value of a is 5



③ When the value of a is 11



Interesting Simulation II (Grapes)

2.11.2024
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1 5 3 straight lines passing through one point

(1) Exam question 15

When three straight line ①: $2x+y+3=0$, ②: $x-y+6=0$, ③: $ax+y+24=0$ intersect at one point , find the value of the constant a.

(2) Experimental result (Grapes version simulation)

【Experiment day】

February 11 , 2024

【PC used】

Lavie NX850/N

【GRAPES used】

GRAPES 7.84

【Script used】

Self-made file

『examquestion15.gps』

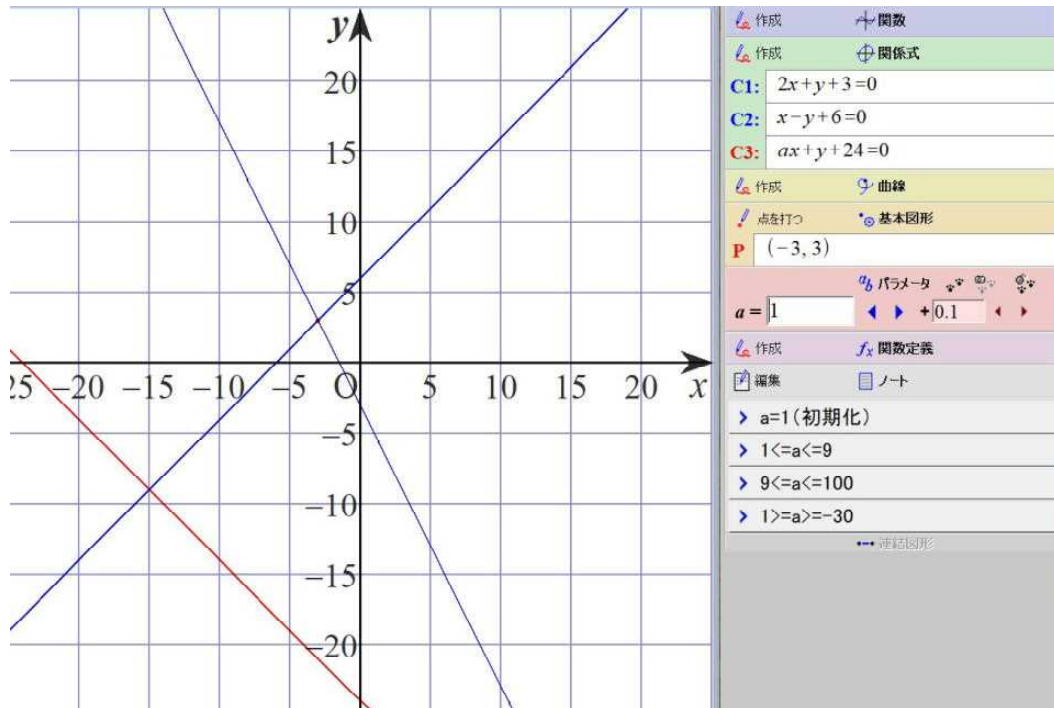
【Consideration】

I varied the value of a from -30 to 100 and observed the intersection of three straight lines.

When $a=9$, the three straight lines intersect at one point.

(Find the intersection of the two straight lines ① and ②. Since the coordinates of this intersections , $x=-3$, $y=3$ satisfy the straight line equation $ax+y+24=0$, we can find $a=9$ by substituting.)

① When the value of a is 1



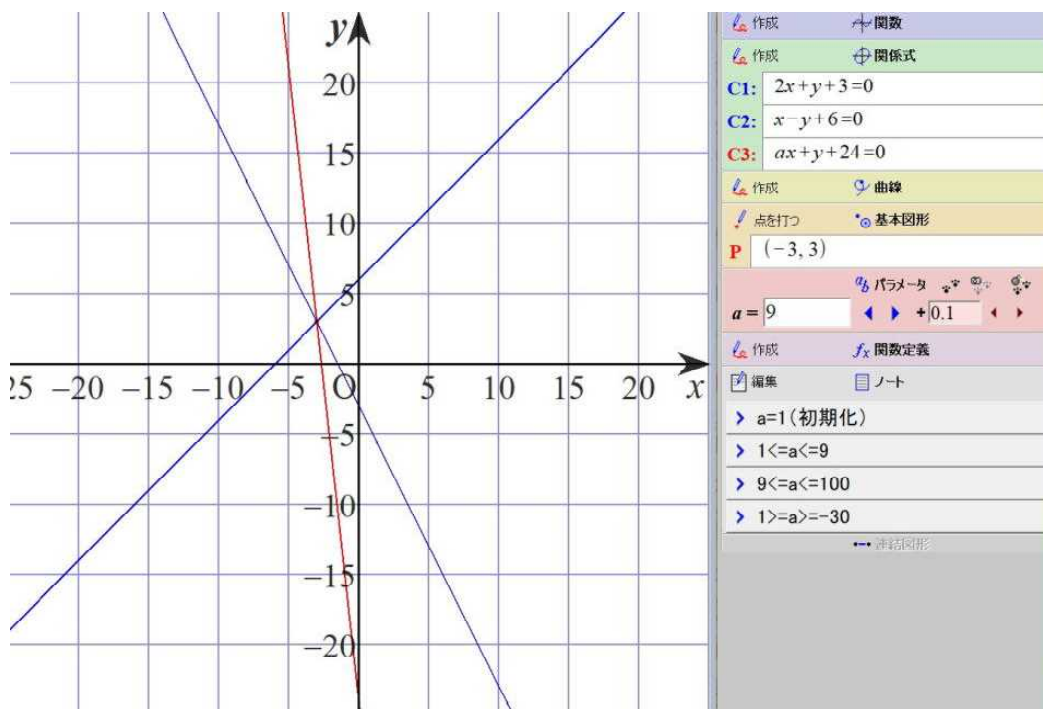
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2.11.2024
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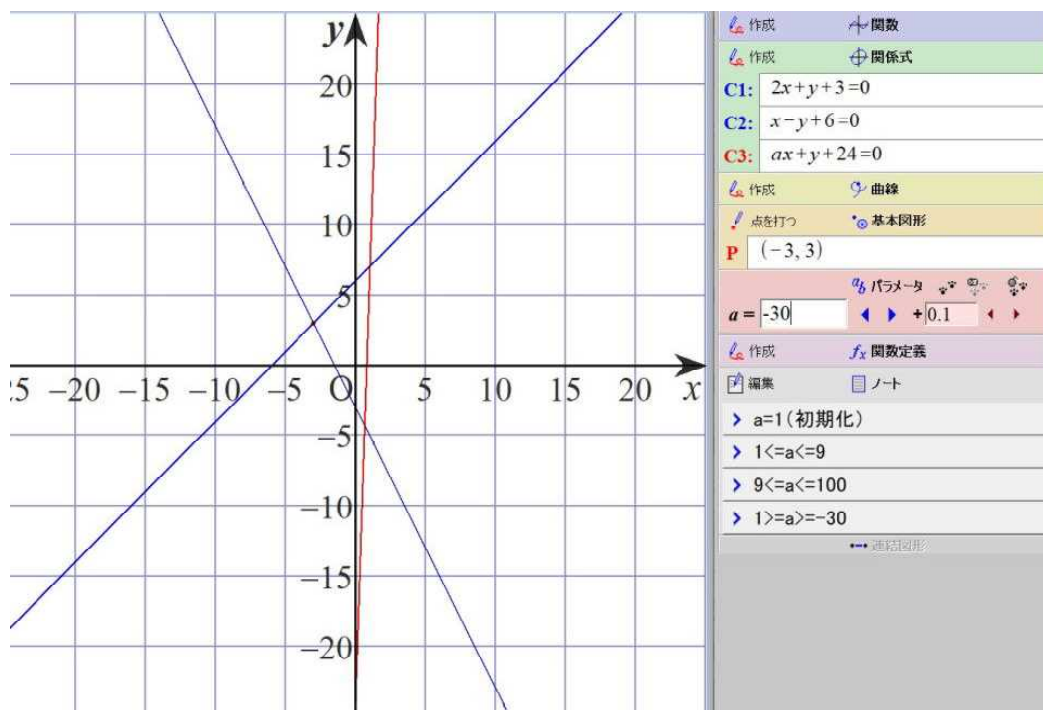
1 5 3 straight lines passing through one point

(2) Experimental result (Grapes version simulation)

② When the value of a is 9



③ When the value of a is -30



Interesting Simulation II (Grapes)

2.12.2024
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1 6 A circle tangent to both the x and y axis

(1) Exam question 16

Find the equation of circle ① whose center is on the straight line $y=-4x+5$ and is tangent to both the x-axis and y-axis.

(2) Experimental result (Grapes version simulation)

【Experiment day】

February 12, 2024

【PC used】

Lavie NX850/N

【GRAPES used】

GRAPES 7.84

【Script used】

Self-made file

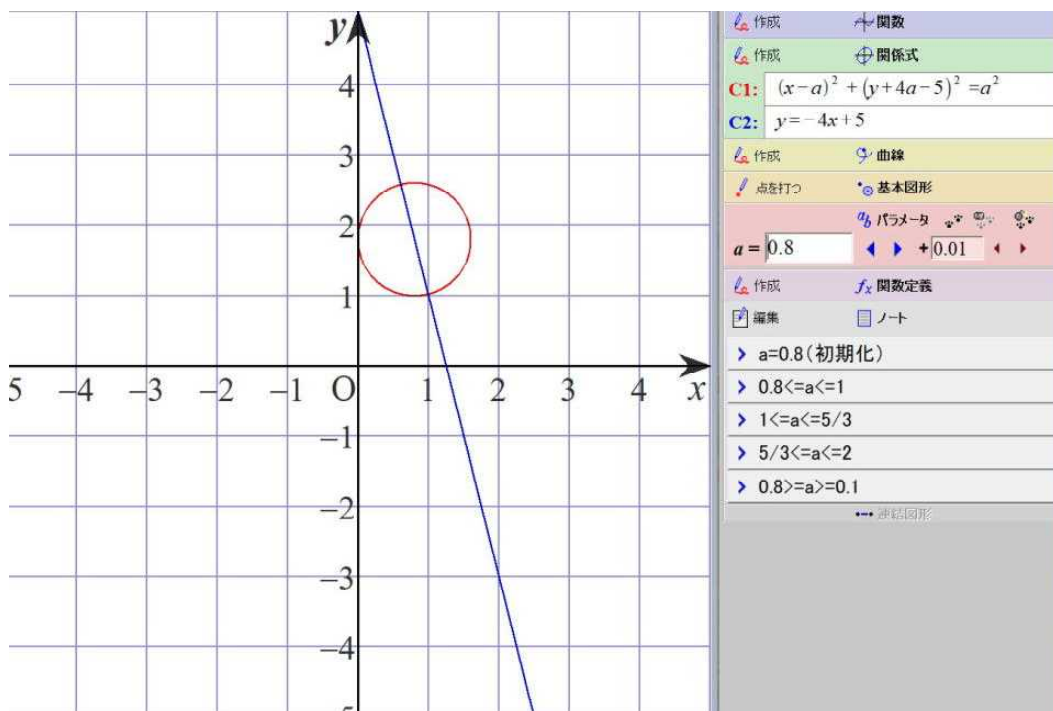
『examquestion16.gps』

【Consideration】

I varied the value of a from 0.1 to 2 and observed the positional relationship between the circle ①, the x-axis, and the y-axis. When $a=1$, circle ① is in the first quadrant and touches both the x-axis and y-axis. When $a=5/3$, circle ① is in the 4th quadrant and touches both the x-axis and y-axis. (The coordinates of the center of the circle in the first quadrant are $(a, -4a+5)$, and since it touches both the x-axis and y-axis, $a=-4a+5$ holds true. Therefore, $a=1$ can be found.) (The coordinates of the center of the circle in the 4th quadrant are $(a, -4a+5)$, and since it touches both the x-axis and y-axis, $a=-(-4a+5)$ holds true. Therefore, $a=5/3$ can be found.)

$$\text{Therefore, } (x-1)^2 + (y-1)^2 = 1, \quad \left(x - \frac{5}{3}\right)^2 + \left(y + \frac{5}{3}\right)^2 = \frac{25}{9}.$$

① When the value of a is 0.8



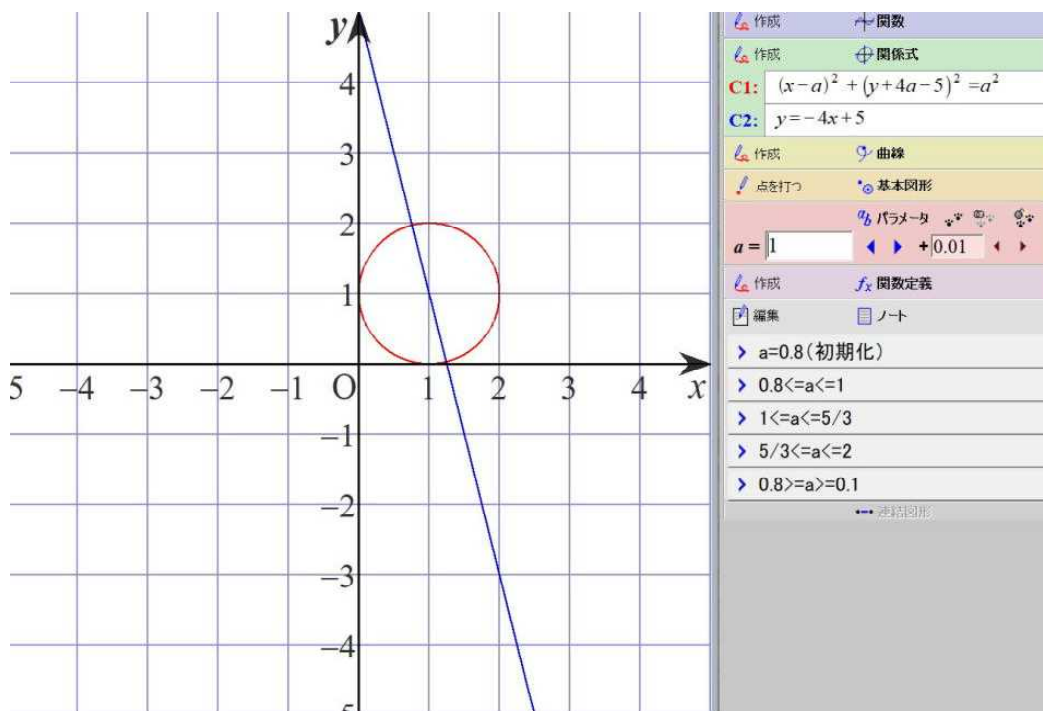
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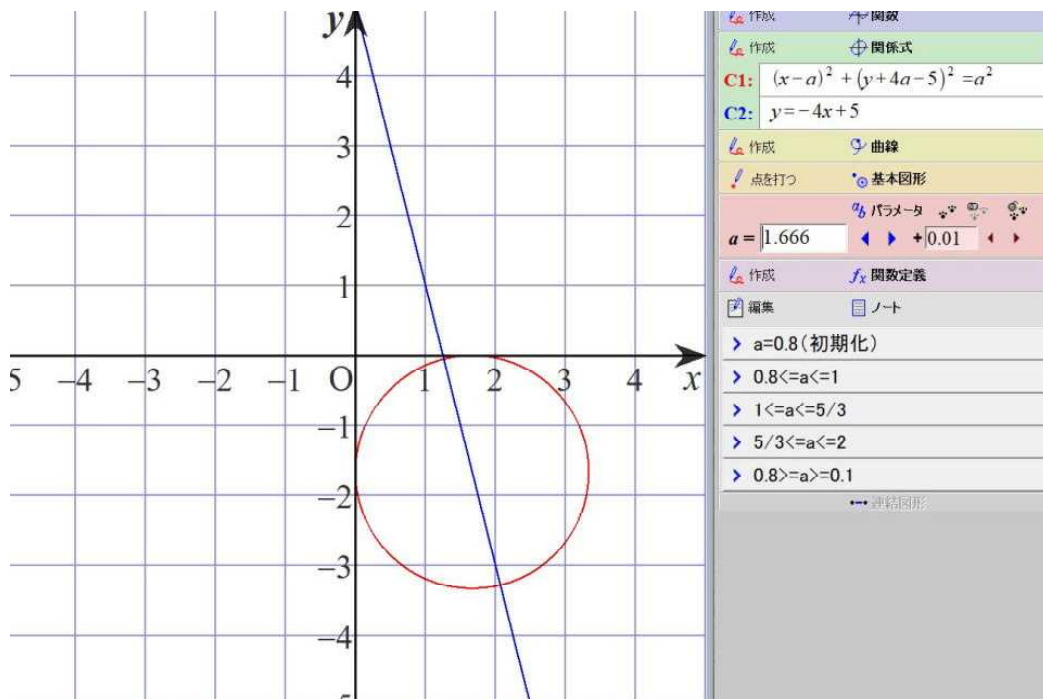
1 6 A circle tangent to both the x and y axis

(2) Experimental result (Grapes version simulation)

② When the value of a is 1



③ When the value of a is 5/3



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1 7 A straight line that passes a fixed point and touches a circle

(1) Exam question 17

Find the equation of the straight line L that passes through the point (3,1) and is tangent to the circle C: $x^2 + y^2 = 2$, and the coordinates of the point of contact at that time.

(2) Experimental result (Grapes version simulation)

【Experiment day】

February 12, 2024

【PC used】

Lavie NX850/N

【GRAPES used】

GRAPES 7.84

【Script used】

Self-made file

『examquestion17.gps』

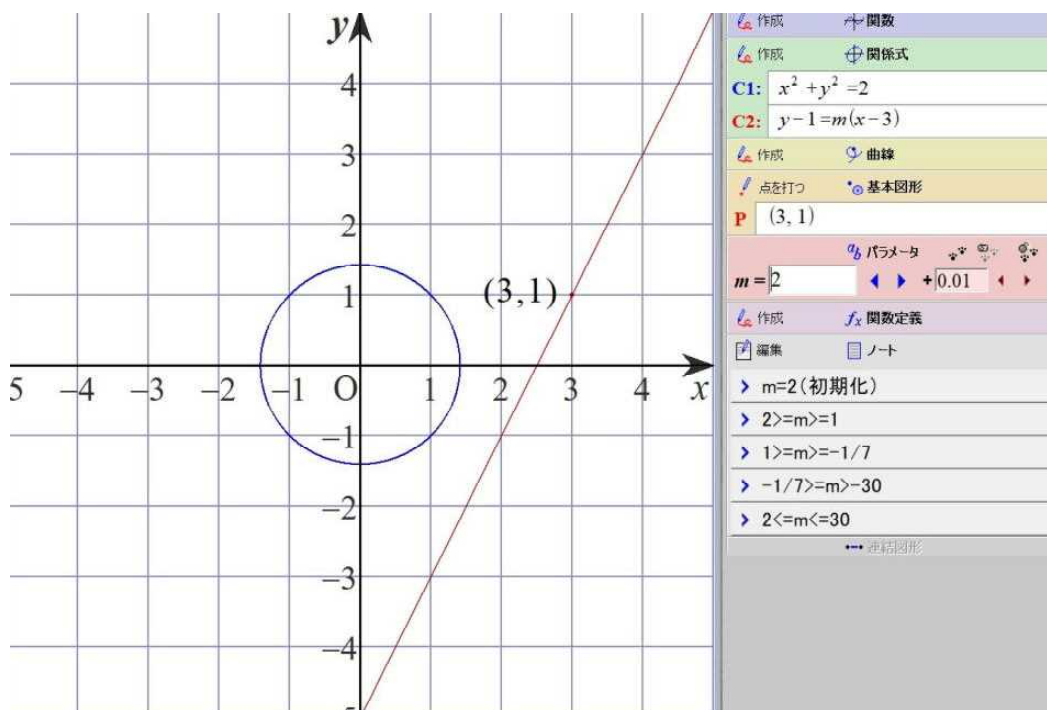
【Consideration】

I varied the value of m from -30 to 30 and observed the positional relationship between circle C and straight line L. When $m=1$, straight line L touches circle C. When $m=1/7$, straight line L touches circle C.

(If the coordinates of the point of contact are (x_1, y_1) , the equation of the tangent to circle C is $L_1: x_1x + y_1y = 2$. Since the tangent L_1 passes through (3,1), it satisfies ①: $3x_1 + y_1 = 2$. Also, since (x_1, y_1) is a point on circle C, it satisfies ②: $x_1^2 + y_1^2 = 2$. If you solve ① and ② simultaneously, you will find $x_1=1, y_1=-1$, and $x_1=1/5, y_1=7/5$.)

Therefore, $x - y = 2$, $x + 7y = 10$.

① When the value of m is 2



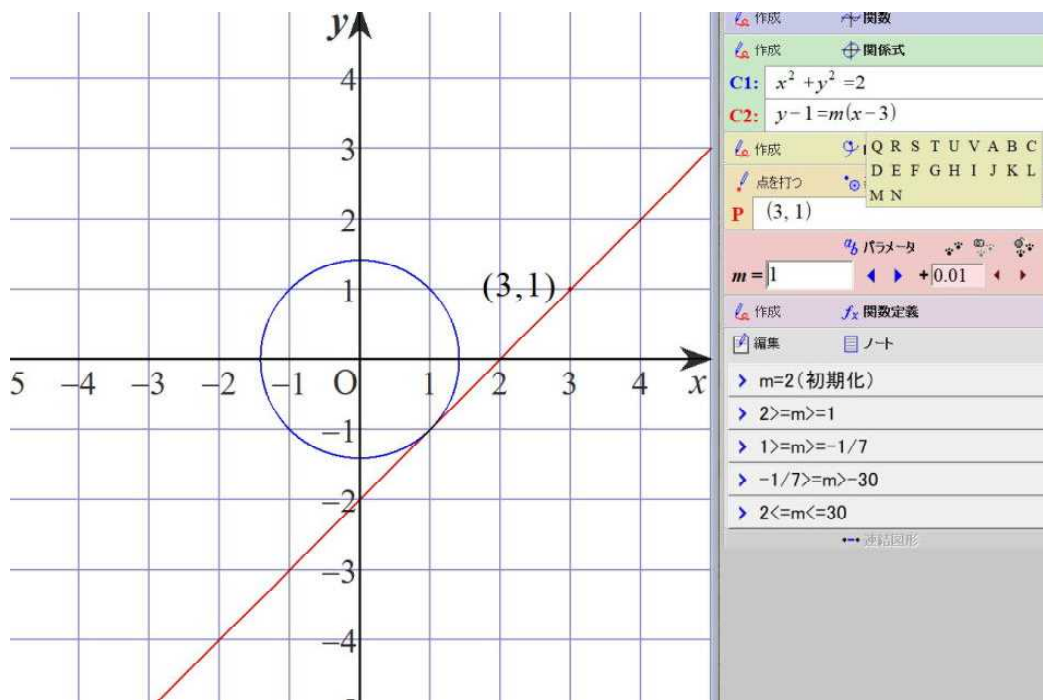
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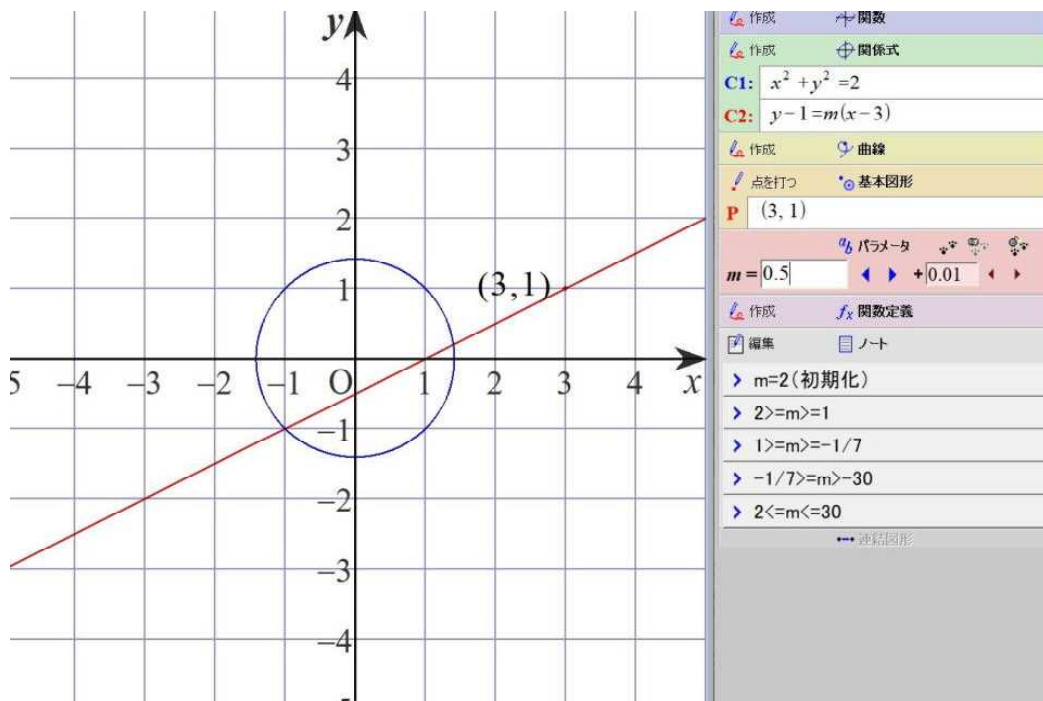
1 7 A straight line that passes a fixed point and touches a circle

(2) Experimental result (Grapes version simulation)

② When the value of m is 1



③ When the value of m is 0.5



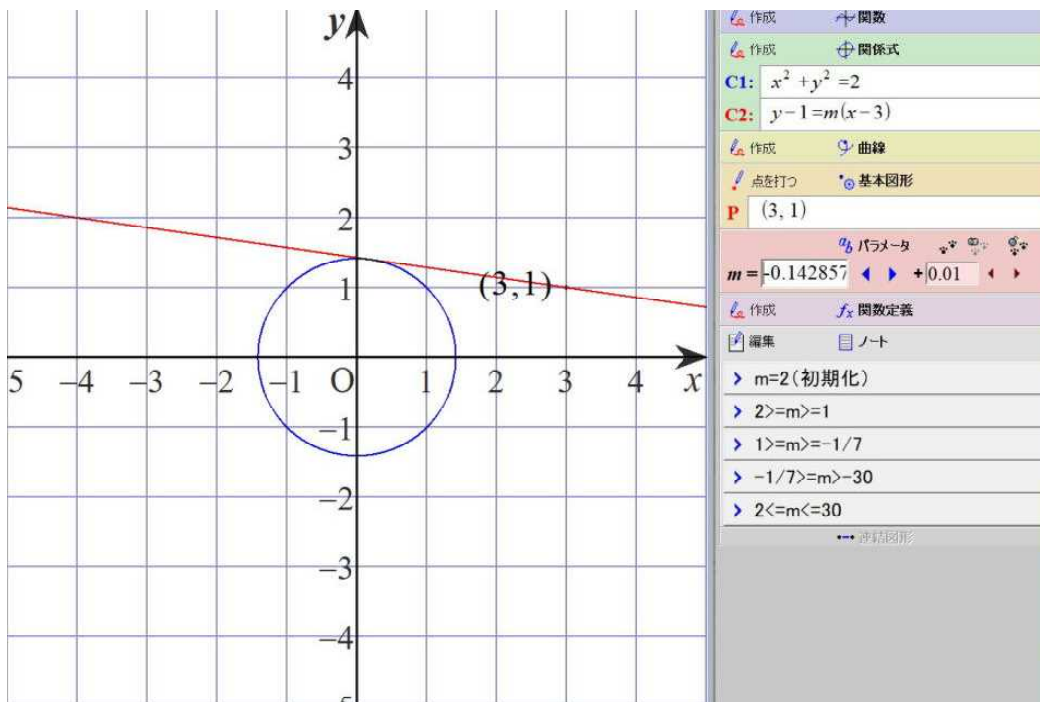
Interesting Simulation II (Grapes)

2.12.2024
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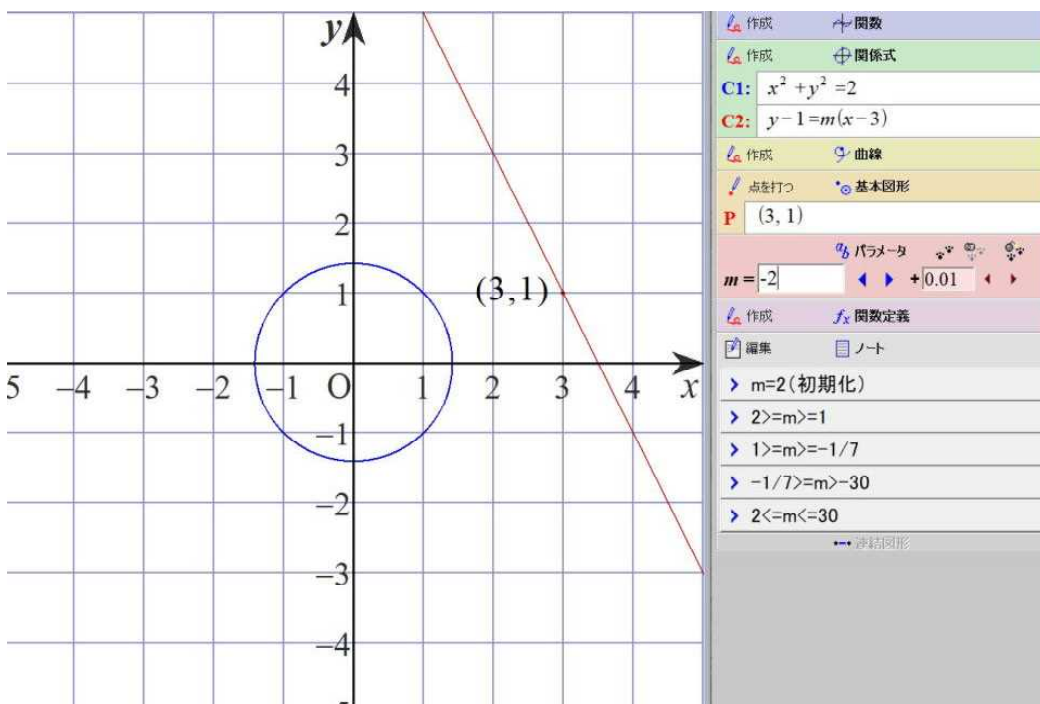
1.7 A straight line that passes a fixed point and touches a circle

(2) Experimental result (Grapes version simulation)

④ When the value of m is $-1/7$



⑤ When the value of m is -2



Interesting Simulation II (Grapes)

2.13.2024
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1 8 Minimum value of quadratic function

(1) Exam question 18

Find the minimum value of the quadratic function $y=x^2-2x$ ($a \leq x \leq a+1$).

(2) Experimental result (Grapes version simulation)

【Experiment day】

February 13, 2024

【PC used】

Lavie NX850/N

【GRAPES used】

GRAPES 7.84

【Script used】

Self-made file

『examquestion18.gps』

【Consideration】

I varied the value of a from -1.3 to 3 and observed the minimum value of the quadratic function.

When $a < 0$, the minimum value is obtained when $x = a + 1$.

When $0 \leq a \leq 1$, the minimum value is obtained at the vertex.

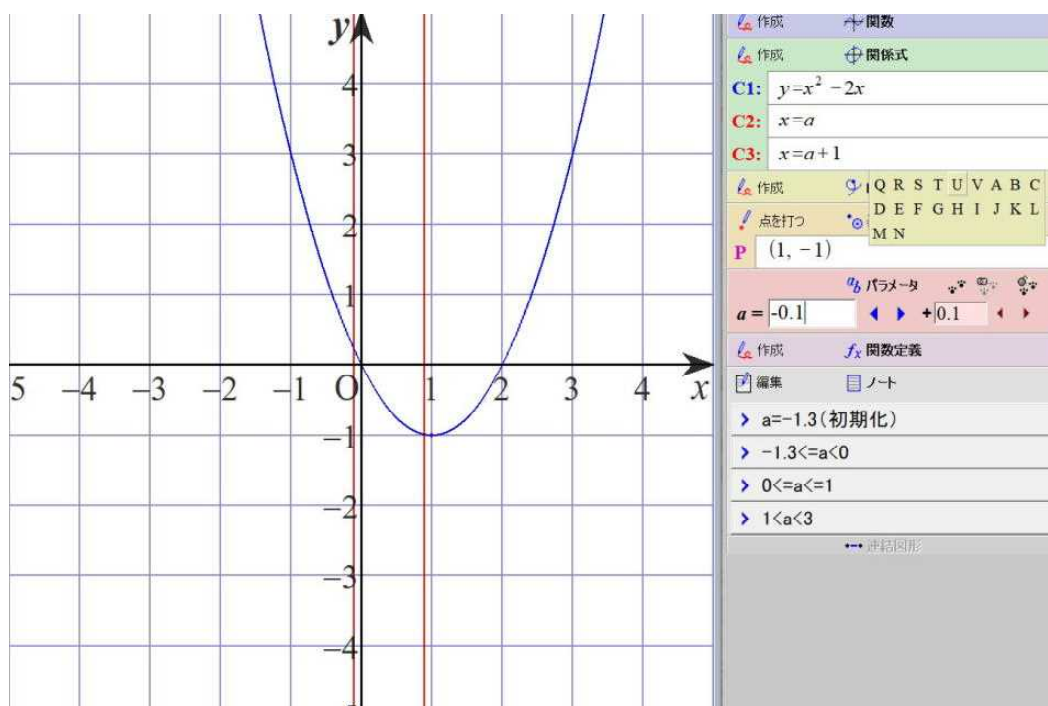
When $1 < a$, the minimum value is obtained when $x = a$.

Therefore, when $a < 0$, when $x = a + 1$, the minimum value is $a^2 - 1$.

When $0 \leq a \leq 1$, the minimum value is -1 at the vertex.

When $1 < a$, when $x = a$, the minimum value is $a^2 - 2a$.

① When the value of a is -0.1



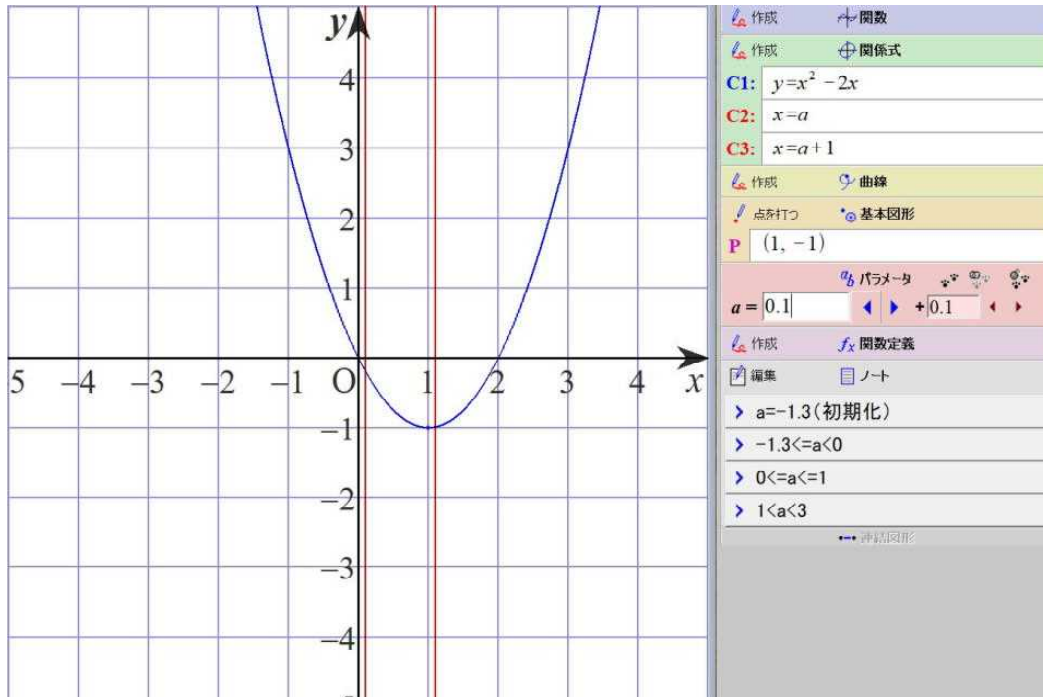
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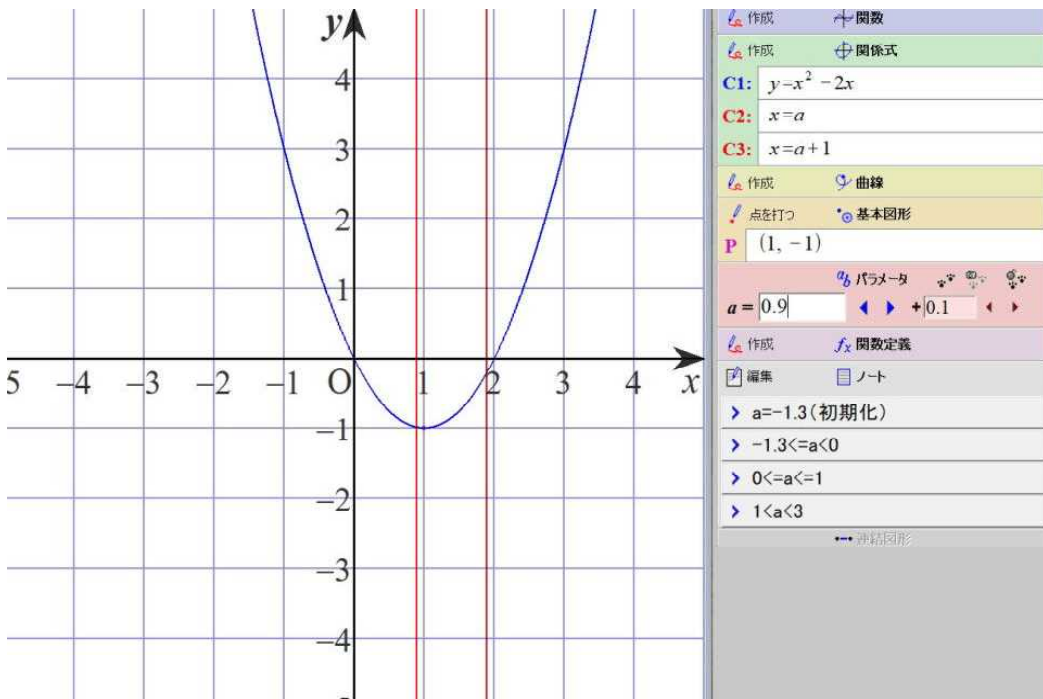
1 8 Minimum value of quadratic function

(2) Experimental result (Grapes version simulation)

② When the value of a is 0.1



③ When the value of a is 0.9



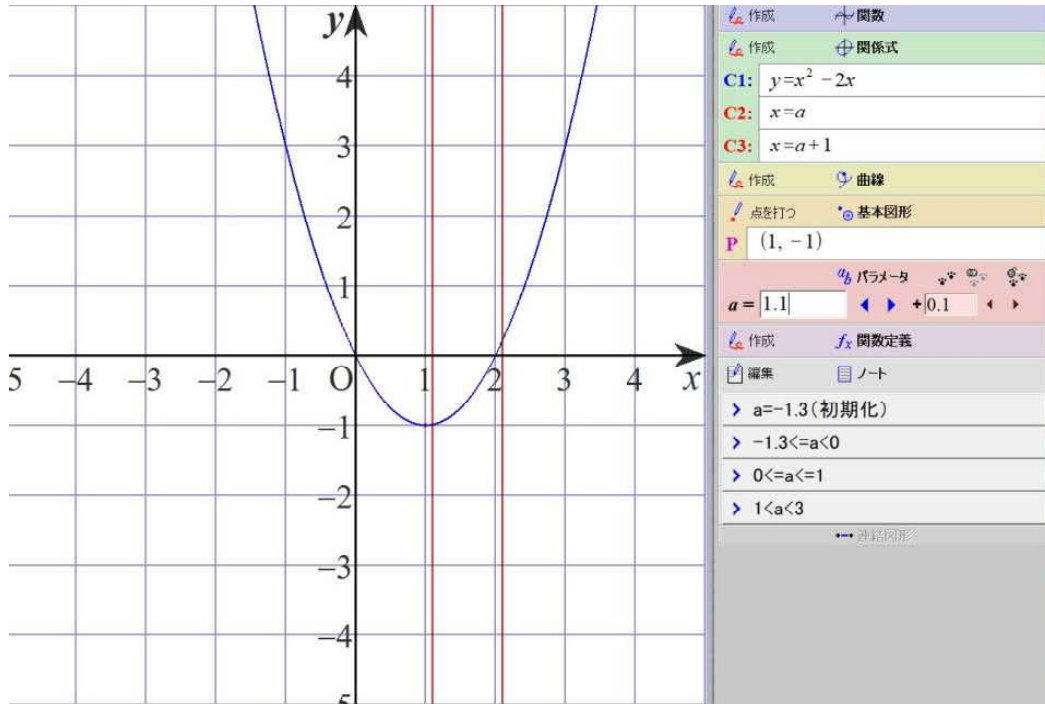
Interesting Simulation II (Grapes)

2.13.2024
Sohun

1 8 Minimum value of quadratic function

(2) Experimental result (Grapes version simulation)

④ When the value of a is 1.1



Interesting Simulation II (Grapes)

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1 9 Proof of cubic inequality

(1) Exam question 19

For all x with $x > 0$, find the range of values of constant a such that the cubic inequality $x^3 - 3a^2x + 2 > 0$ holds true.

(2) Experimental result (Grapes version simulation)

【Experiment day】

February 14, 2024

【PC used】

Lavie NX850/N

【GRAPES used】

GRAPES 7.84

【Script used】

Self-made file

『examquestion19.gps』

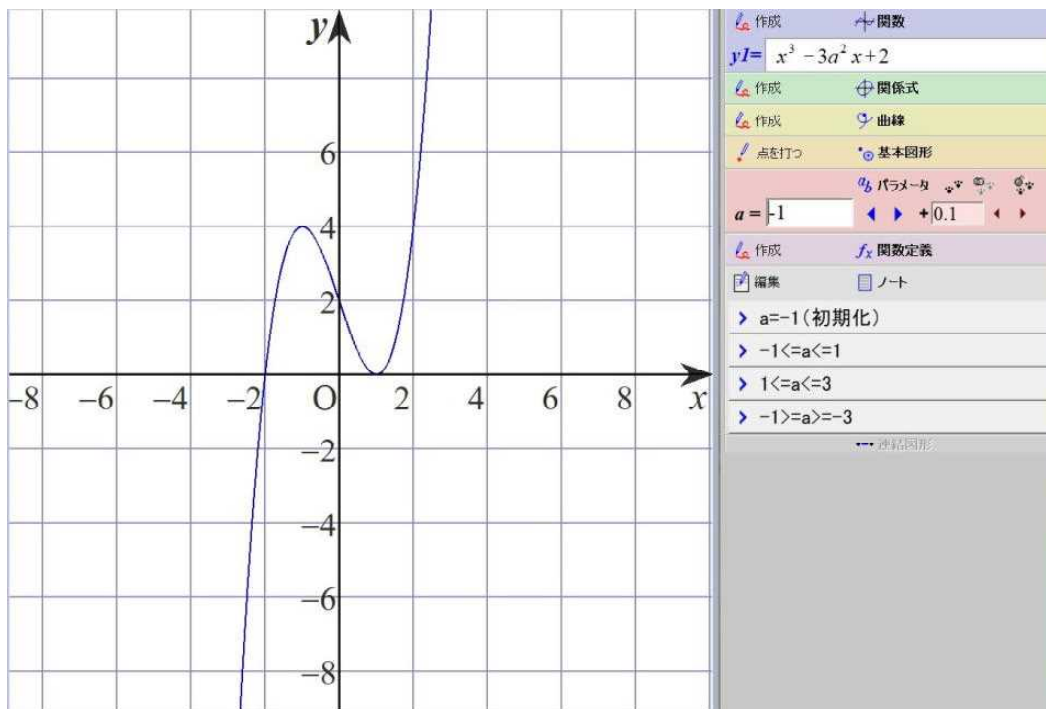
【Consideration】

I varied the value of a from -3 to 3 and observed the graph of the cubic function $y = x^3 - 3a^2x + 2$. When $-1 < a < 1$, the graph of the cubic function $y = x^3 - 3a^2x + 2$ is above the x -axis for all $x \geq 0$.

(Since the local minimum value (minimum value) of the cubic function $y = x^3 - 3a^2x + 2$ when $x \geq 0$ is greater than 0 , we can find $-1 < a < 1$.)

Therefore, $-1 < a < 1$.

① When the value of a is -1



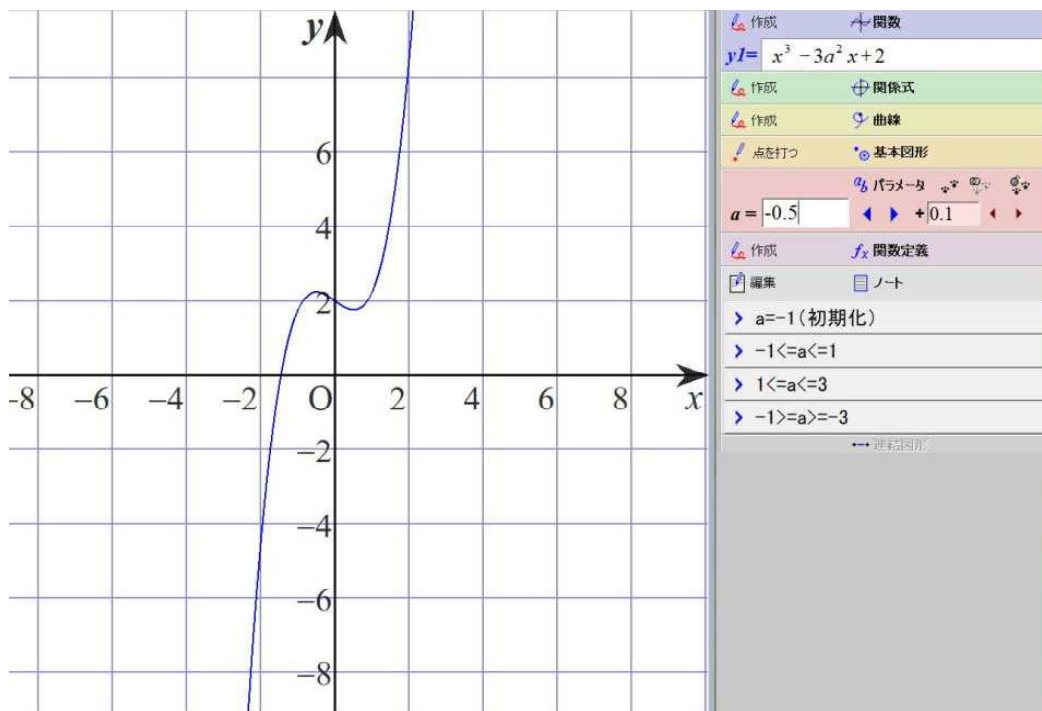
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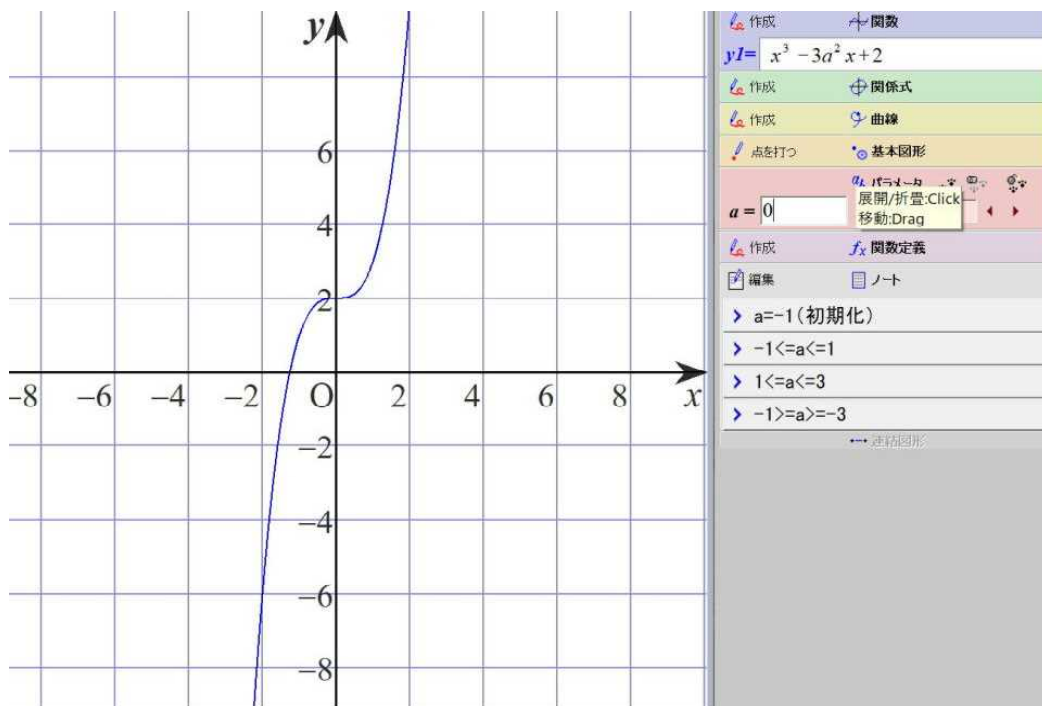
1 9 Proof of cubic inequality

(2) Experimental result (Grapes version simulation)

② When the value of a is -0.5



③ When the value of a is 0



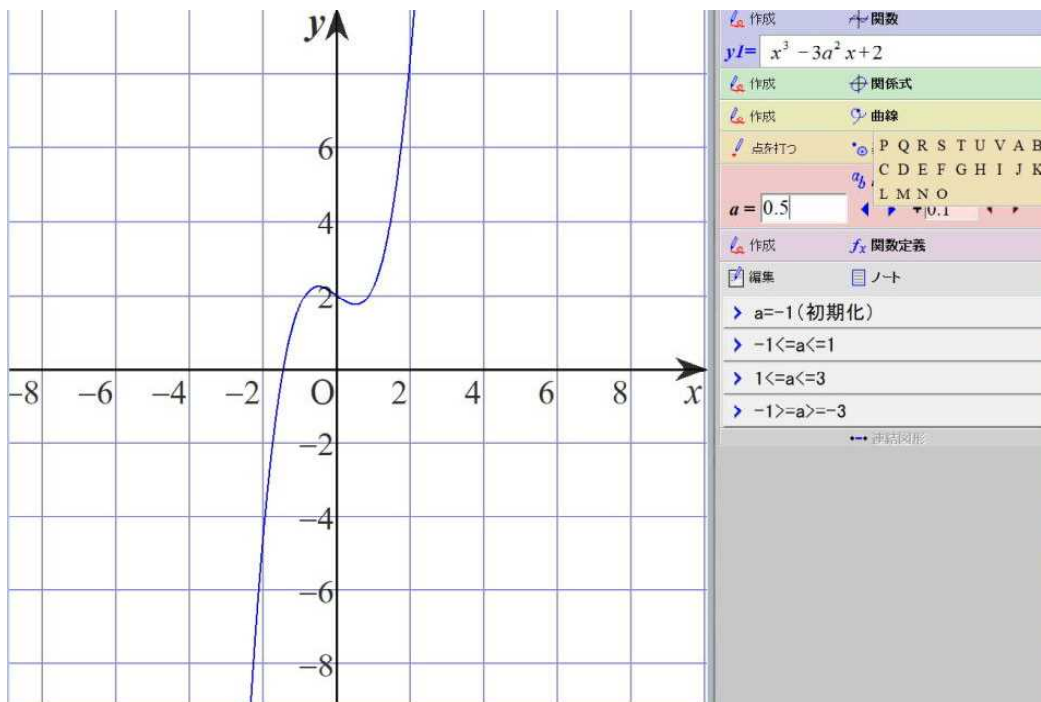
Interesting Simulation II (Grapes)

2.14.2024
Sohun

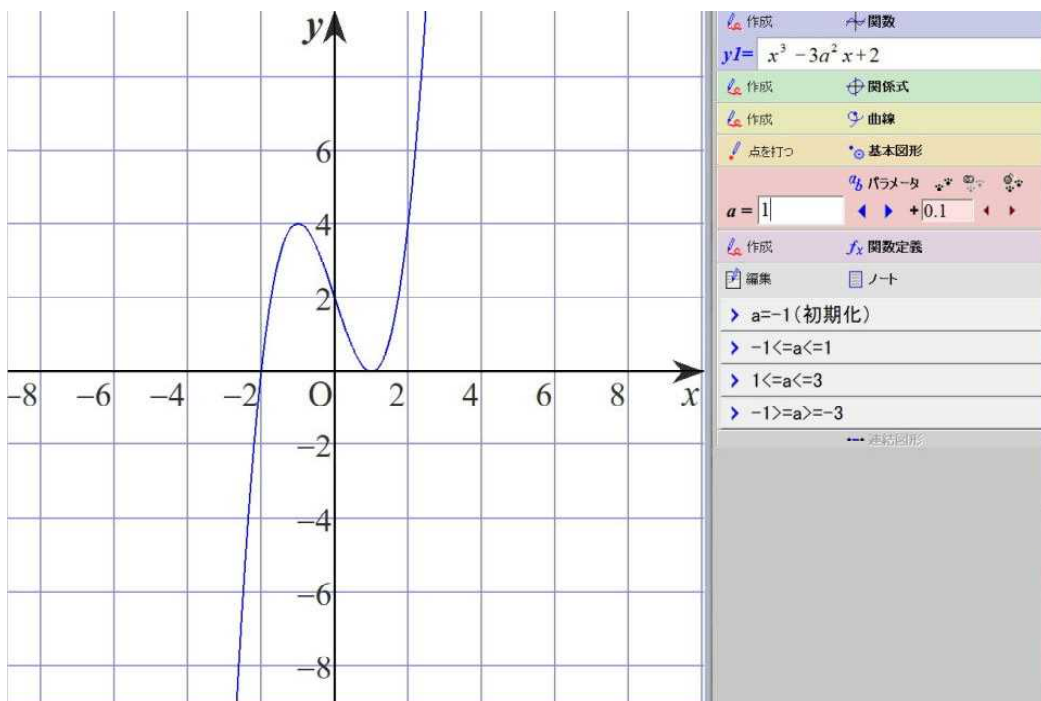
1 9 Proof of cubic inequality

(2) Experimental result (Grapes version simulation)

④ When the value of a is 0.5



⑤ When the value of a is 1



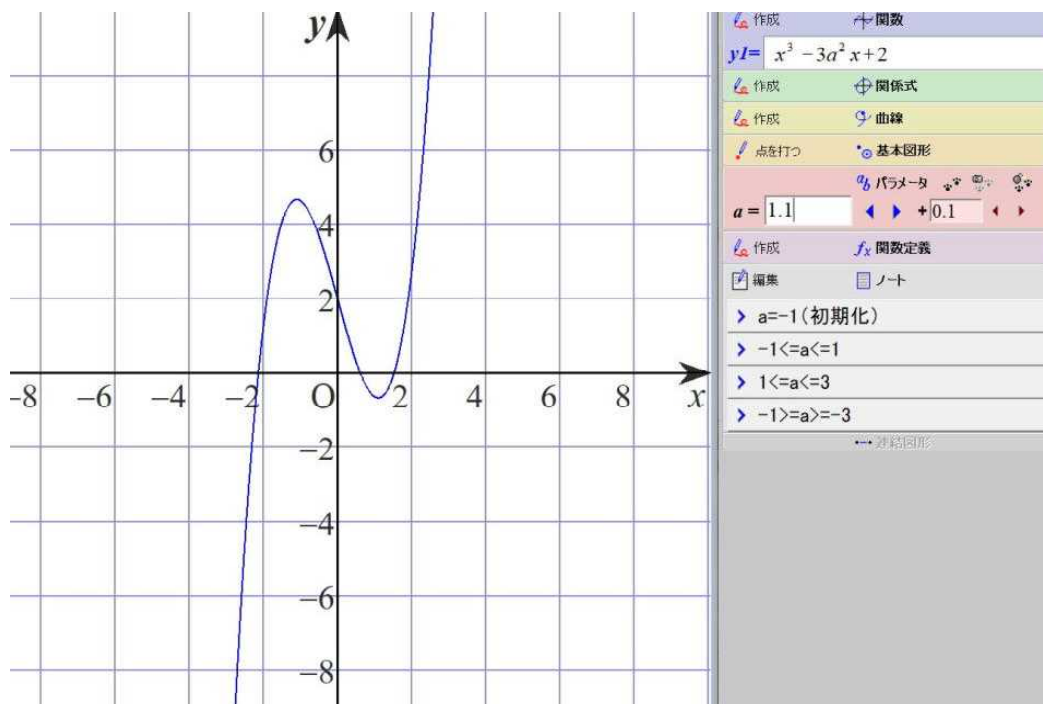
Interesting Simulation II (Grapes)

2.14.2024
Sohun

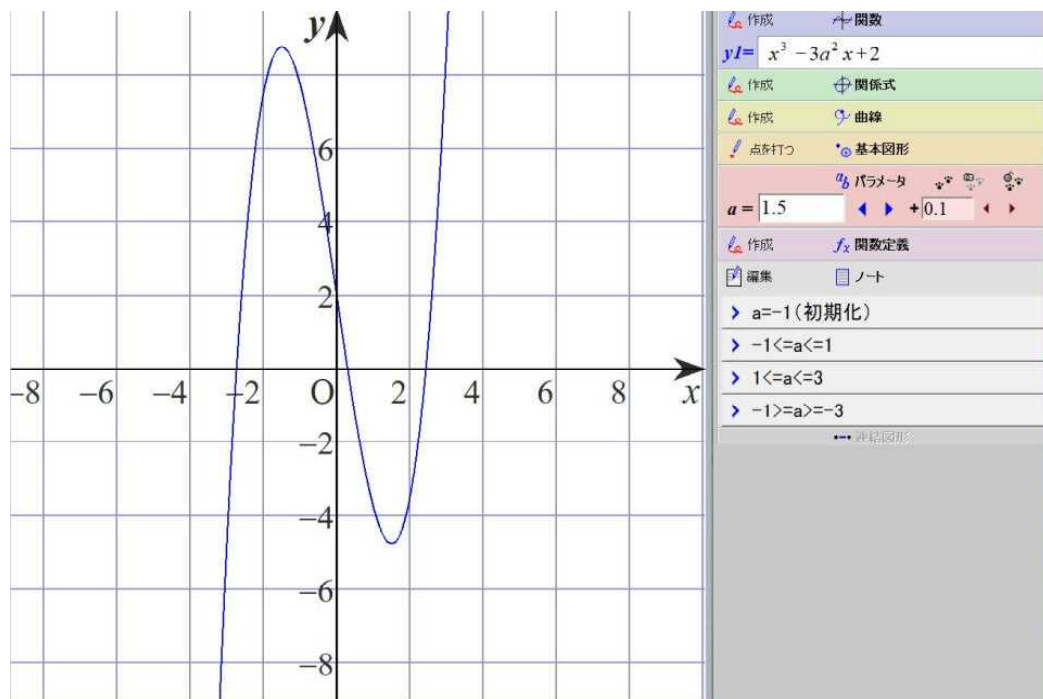
1 9 Proof of cubic inequality

(2) Experimental result (Grapes version simulation)

⑥ When the value of a is 1.1



⑦ When the value of a is 1.5



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2 0 Absolute value graph

(1) Exam question 20

Let a be a constant.

Find the number of solutions to the equation $x^2|x-3|=a$.

(2) Experimental result (Grapes version simulation)

【Experiment day】

February 15, 2024

【PC used】

Lavie NX850/N

【GRAPES used】

GRAPES 7.84

【Script used】

Self-made file

『examquestion20.gps』

【Consideration】

I varied the value of a from -0.9 to 4.8 and observed the graphs of ①: $y=x^2|x-3|$ and ②: $y=a$.

When $a < 0$, there is no common point between graphs ① and ②.

When $a = 0$, the number of common points between graphs ① and ② is 2.

When $0 < a < 4$, the graphs of ① and ② intersect at four points.

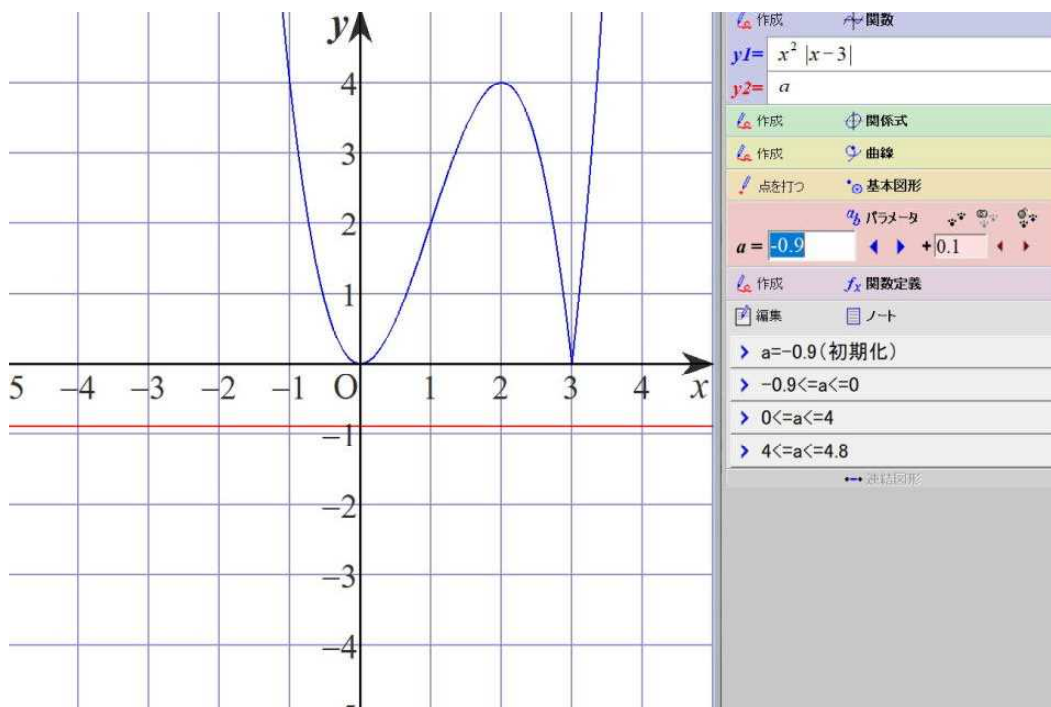
When $a = 4$, the number of common points between graphs ① and ② is 3.

When $a > 4$, the graphs of ① and ② intersect at two points.

Therefore,

When $a < 0$, 0. When $a = 0$, 2. When $0 < a < 4$, 4. When $a = 4$, 3. When $a > 4$, 2.

① When the value of a is -0.9



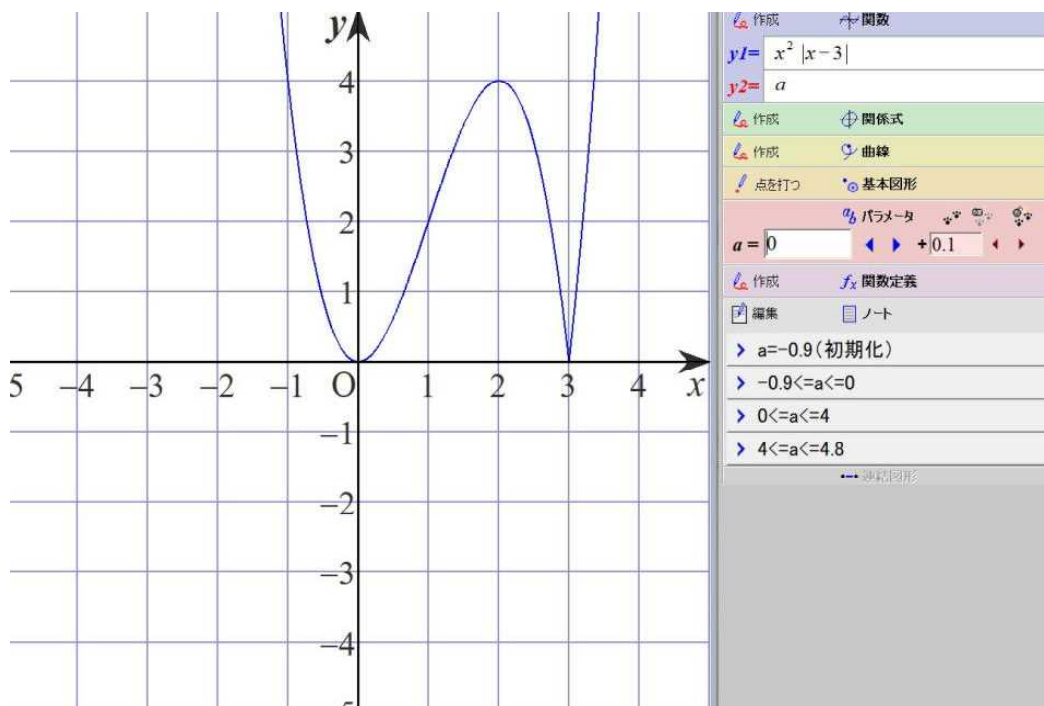
Interesting Simulation II (Grapes)

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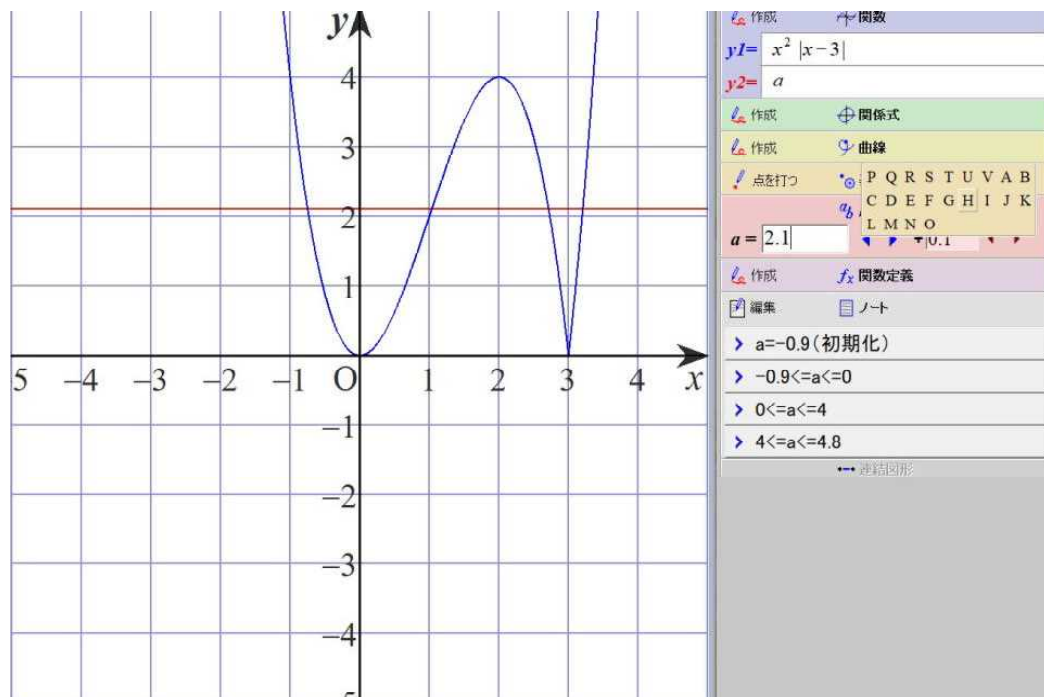
2.0 Absolute value graph

(2) Experimental result (Grapes version simulation)

② When the value of a is 0



③ When the value of a is 2.1



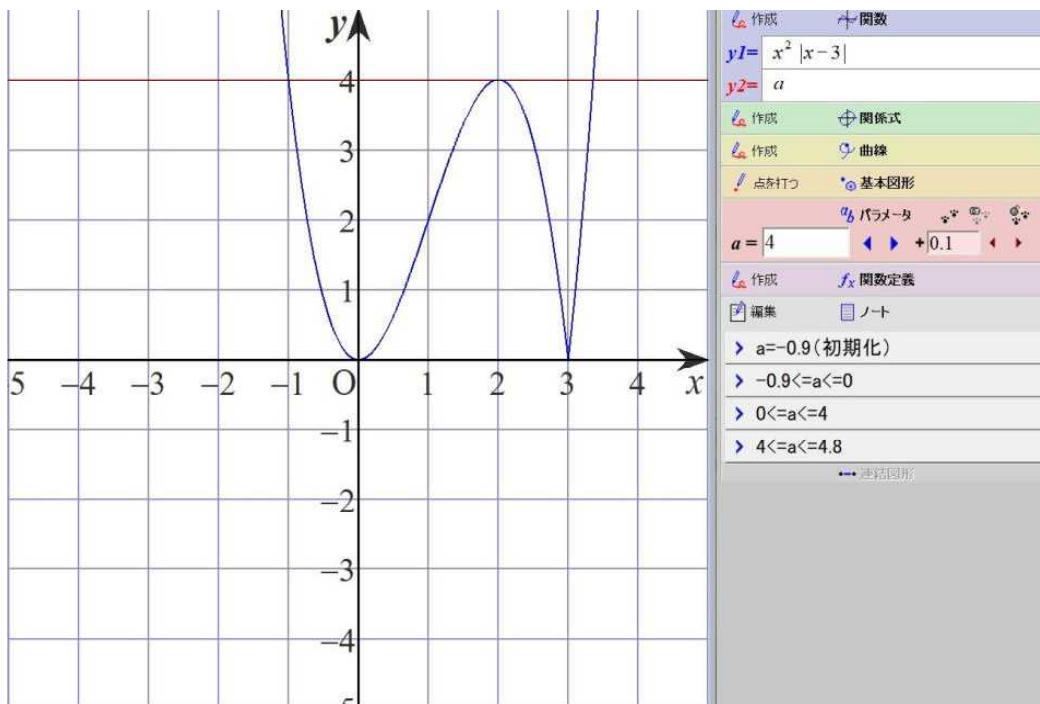
Interesting Simulation II (Grapes)

2.15.2024
Sohun

2 0 Absolute value graph

(2) Experimental result (Grapes version simulation)

④ When the value of a is 4



⑤ When the value of a is 4.3

