

# Interesting Simulation III (Grapes)

1.20.2024  
Sohun

## 1 Tohoku University

### (1) Entrance exam questions

Let  $D$  be the area on the  $xy$ -coordinate plane represented by the inequality  $2y > x + 1 + 3|x - 1|$ .  
For a real number  $a$ , a parabola  $C$  is defined as  $y = x^2 - 2ax + a^2 + a + 2$ .  
At this time, find the range of  $a$  such that all points on  $C$  are points on  $D$ .

### (2) Experimental result (Grapes version simulation)

#### 【Experiment day】

January 20, 2024

#### 【PC used】

VersaPro J VF-F

#### 【GRAPES used】

GRAPES 7.84

#### 【Script used】

Self-made file

『tohoku.gps』

#### 【Consideration】

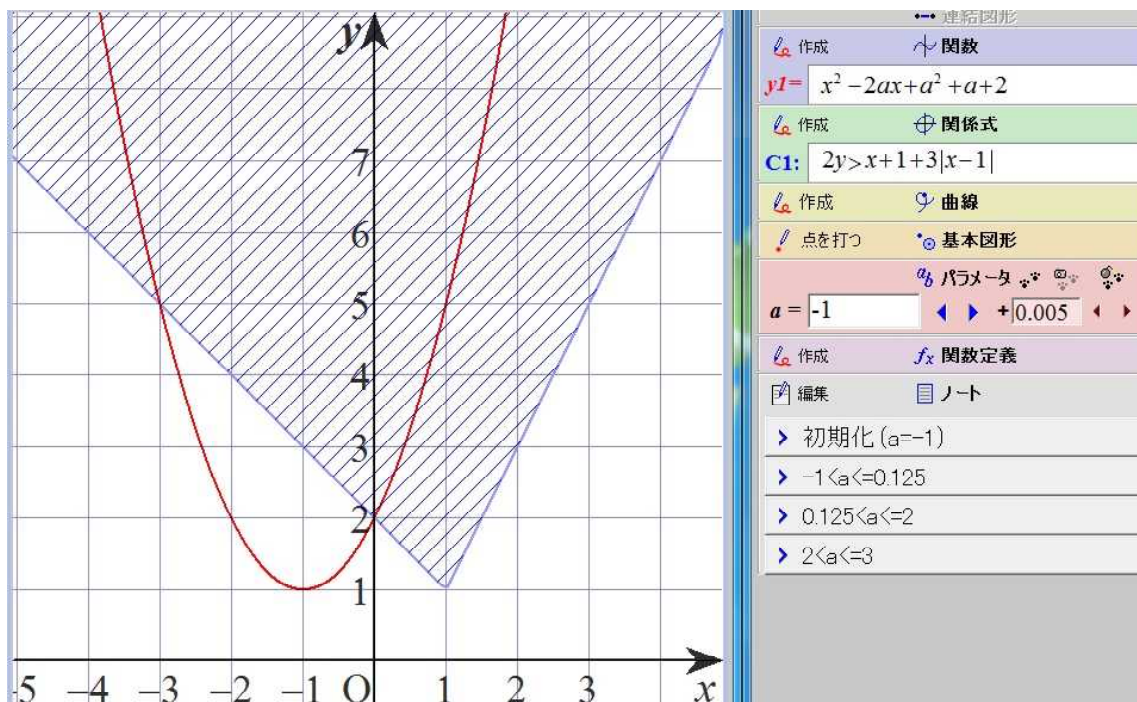
I varied the value of  $a$  from  $-1$  to  $3$  in  $0.005$  increments and observed the parabola.

When the straight line  $y = -x + 2$  ( $x < 1$ ) touches the parabola  $C$ , the simultaneous equations of both have multiple solutions. Therefore, from the multiple solution condition, I calculated and found  $a = 1/8$  (multiple solution  $x = -3/8$ ).

Even when the straight line  $y = 2x - 1$  ( $x \geq 1$ ) touches the parabola  $C$ , both simultaneous equations have multiple solutions. Therefore, from the multiple solution condition, I calculated and found  $a = 2$  (multiple solution  $x = 3$ ).

Therefore, the range of  $a$  such that all points on  $C$  are points on  $D$  is from when the parabola  $C$  touches the straight line  $y = -x + 2$  until it touches the straight line  $y = 2x - 1$ .  $\frac{1}{8} \leq a \leq 2$

#### ① When the value of $a$ is $-1$



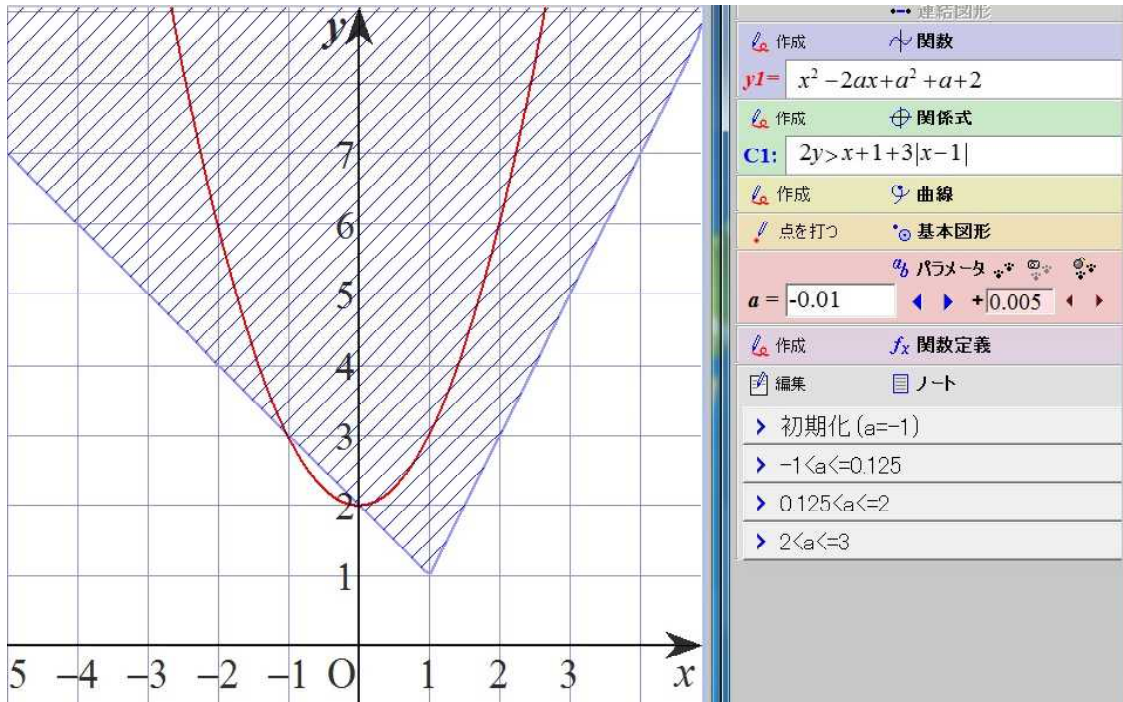
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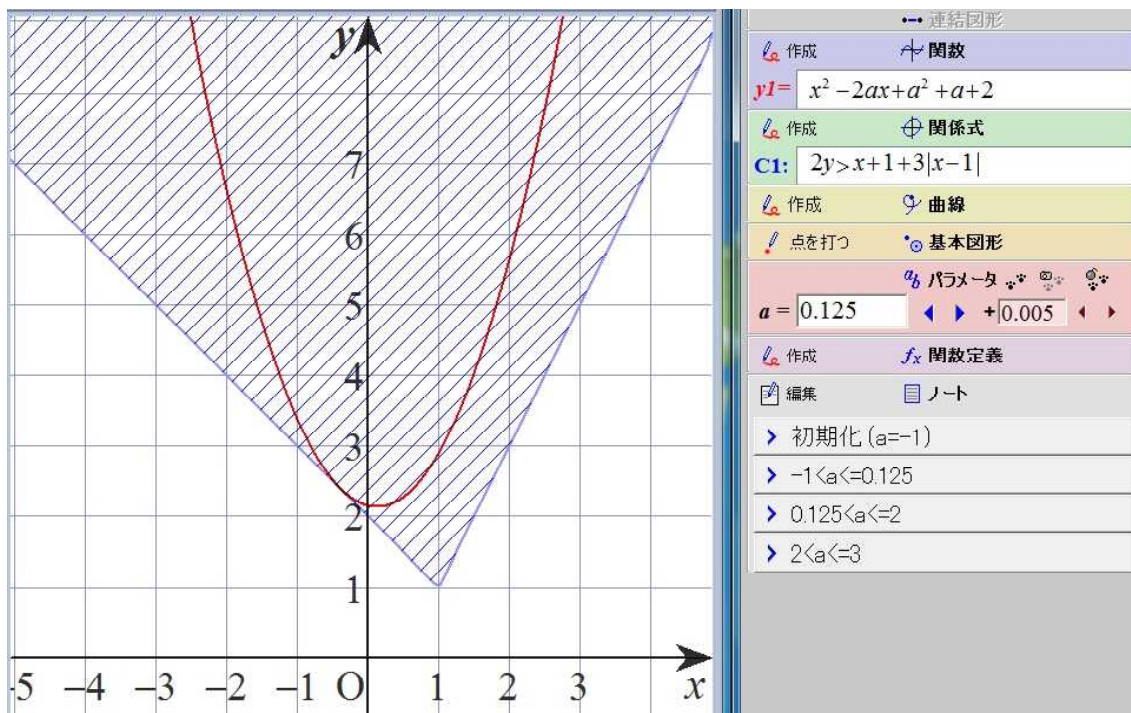
## 1 Tohoku University

(2) Experimental result (Grapes version simulation)

② When the value of a is -0.01



③ When the value of a is 0.125 (1/8)



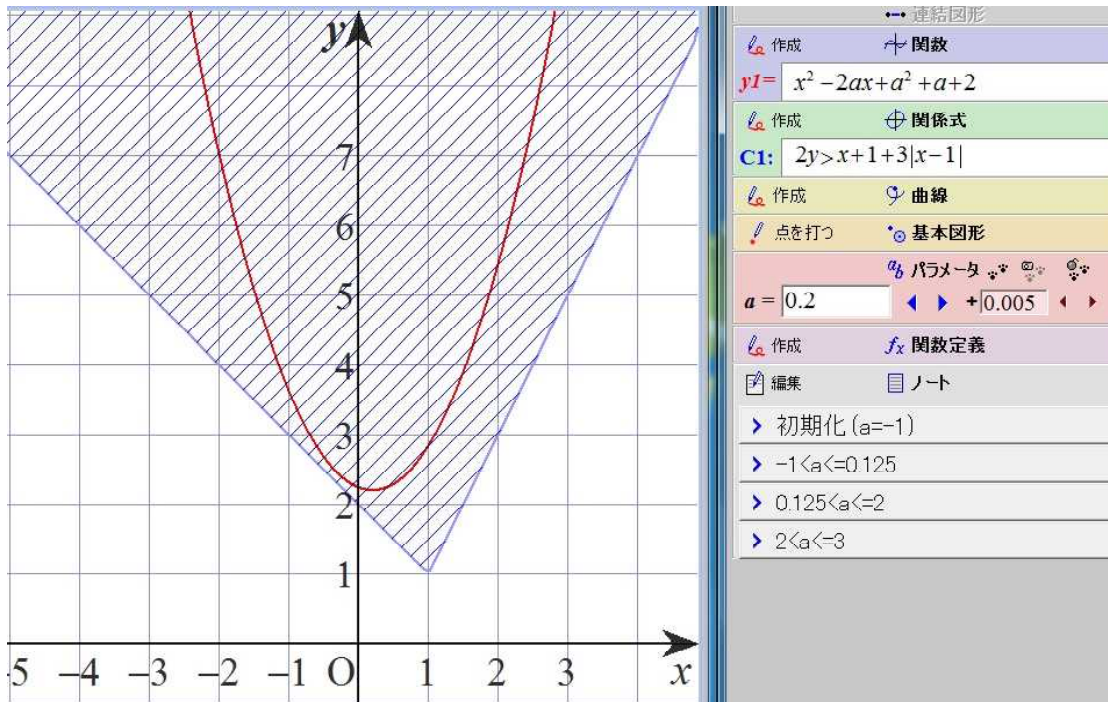
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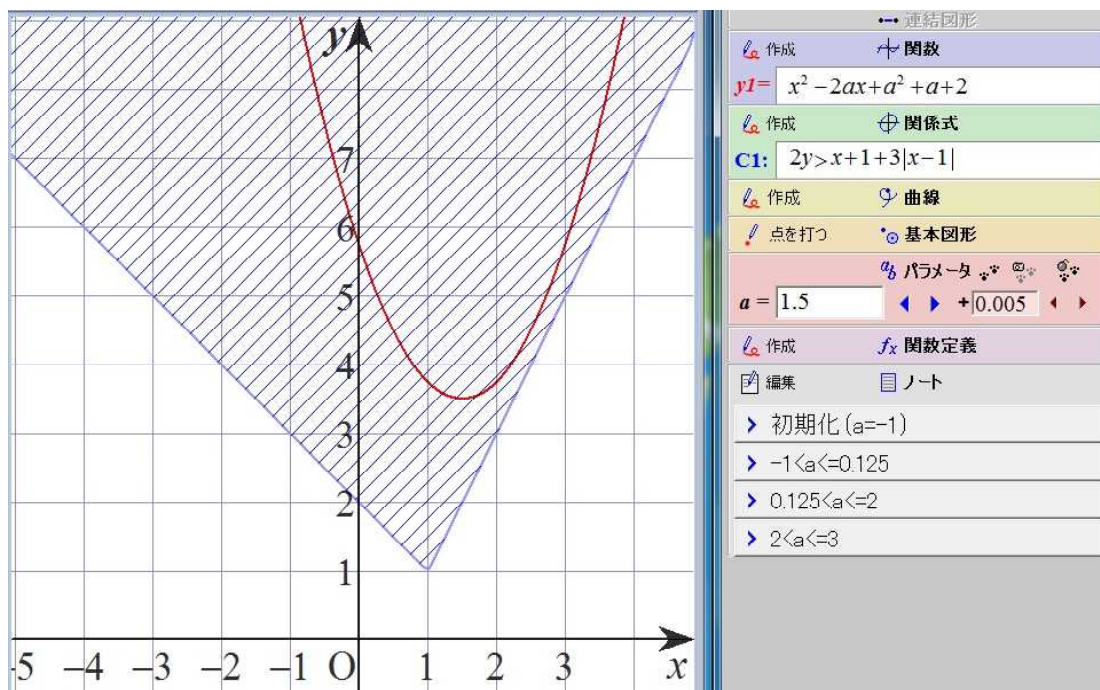
## 1 Tohoku University

(2) Experimental result (Grapes version simulation)

④ When the value of a is 0.2



⑤ When the value of a is 1.5





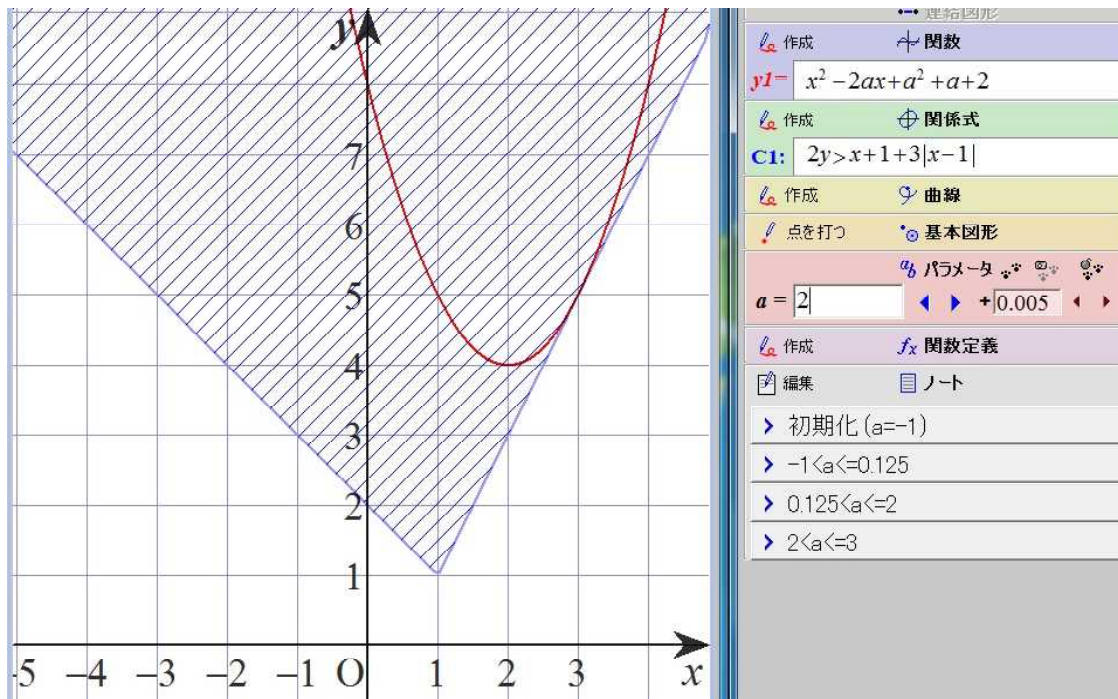
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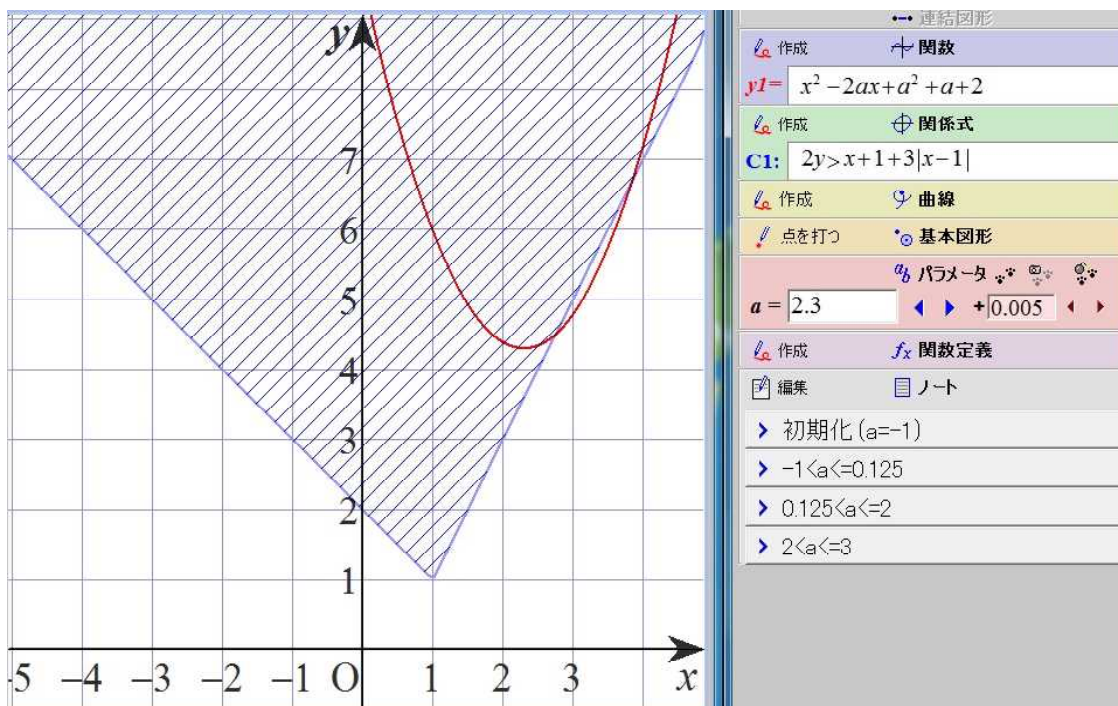
## 1 Tohoku University

(2) Experimental result (Grapes version simulation)

⑥ When the value of a is 2



⑦ When the value of a is 2.3



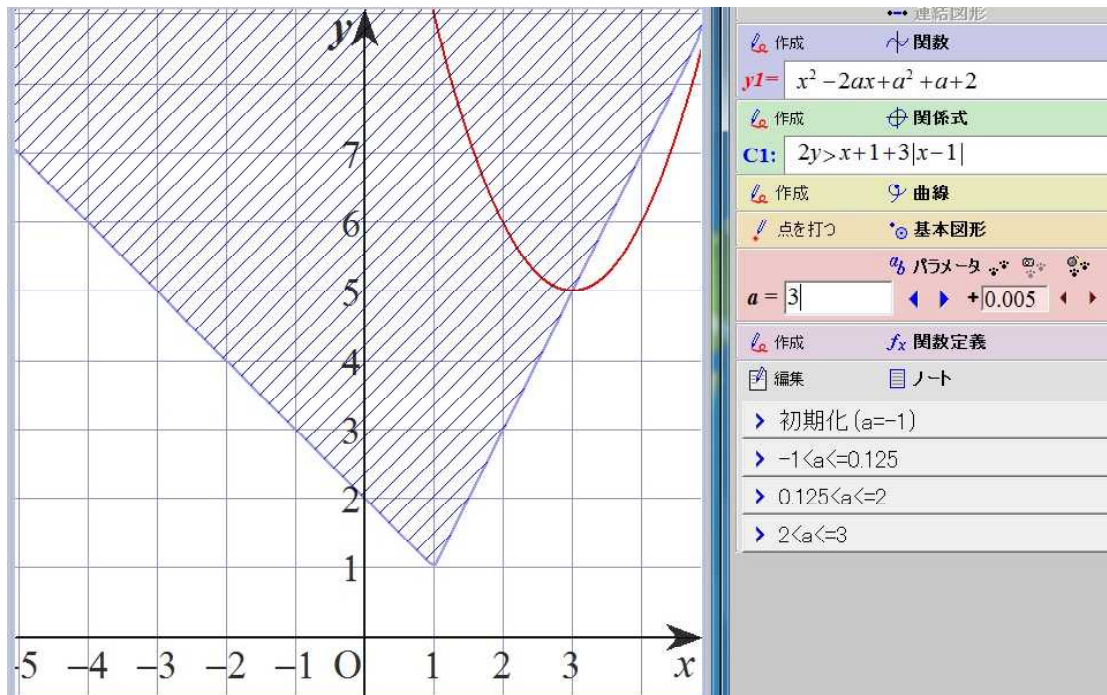
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(2) Experimental result (Grapes version simulation)

⑧ When the value of a is 3



# Interesting Simulation III (Grapes)

1.21.2024  
Sohun

## 2 Kansei Gakuin University

### (1) Entrance exam questions

Let  $C > 0$ .

Consider the number of common points between the parabola  $y = x^2 - 1$  and the circle  $x^2 + y^2 = c^2$  on the  $xy$ -plane.

Find the range of values for  $c$  that maximizes the number of common points.

Also, find the value of  $c$  when the number of common points is an odd number.

### (2) Experimental result (Grapes version simulation)

#### 【Experiment day】

January 21, 2024

#### 【PC used】

Lavie NX850/N

#### 【GRAPES used】

GRAPES 7.84

#### 【Script used】

Self-made file

『kansei.gps』

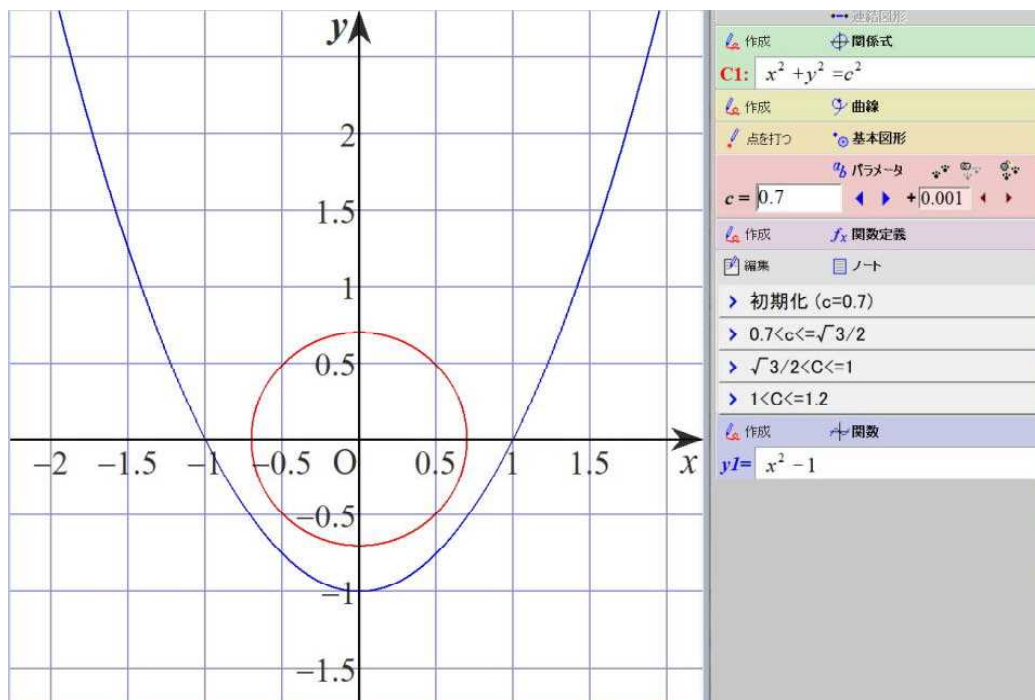
#### 【Consideration】

I varied the radius  $c$  of the circle from 0.7 to 1.2 in 0.001 increments and observed the number of common points. When a circle touches a parabola at two points, both simultaneous equations have multiple solutions. Therefore, from the multiple solution condition, I calculated and found  $c = \sqrt{3}/2$ . Regarding the number of common points, the number of common points is 0 when  $c < \sqrt{3}/2$ , 2 when  $c = \sqrt{3}/2$ , 4 when  $\sqrt{3}/2 < c < 1$ , 3 when  $c = 1$ , and 2 when  $c > 1$ .

Therefore, the maximum number of common points is 4, and the range of  $c$  values at that time is  $\sqrt{3}/2 < c < 1$ .

Also, when the number of common points is an odd number, the number is 3, and the value of  $c$  at that time is  $c = 1$ .

#### ① When the radius $c$ of the circle is 0.7



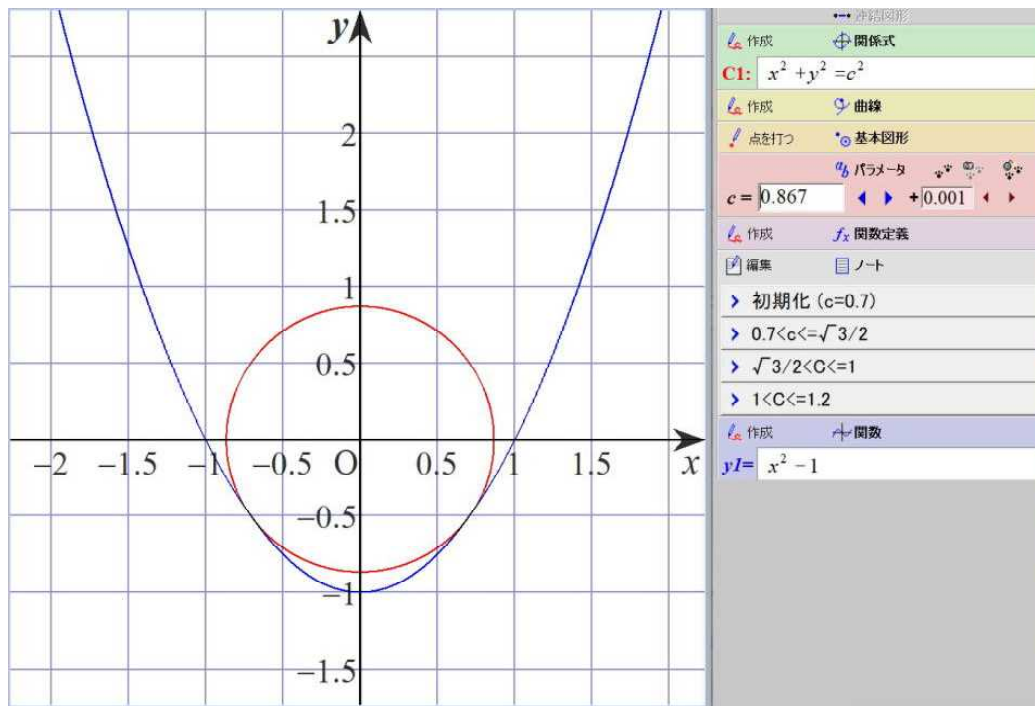
# Interesting Simulation III (Grapes)

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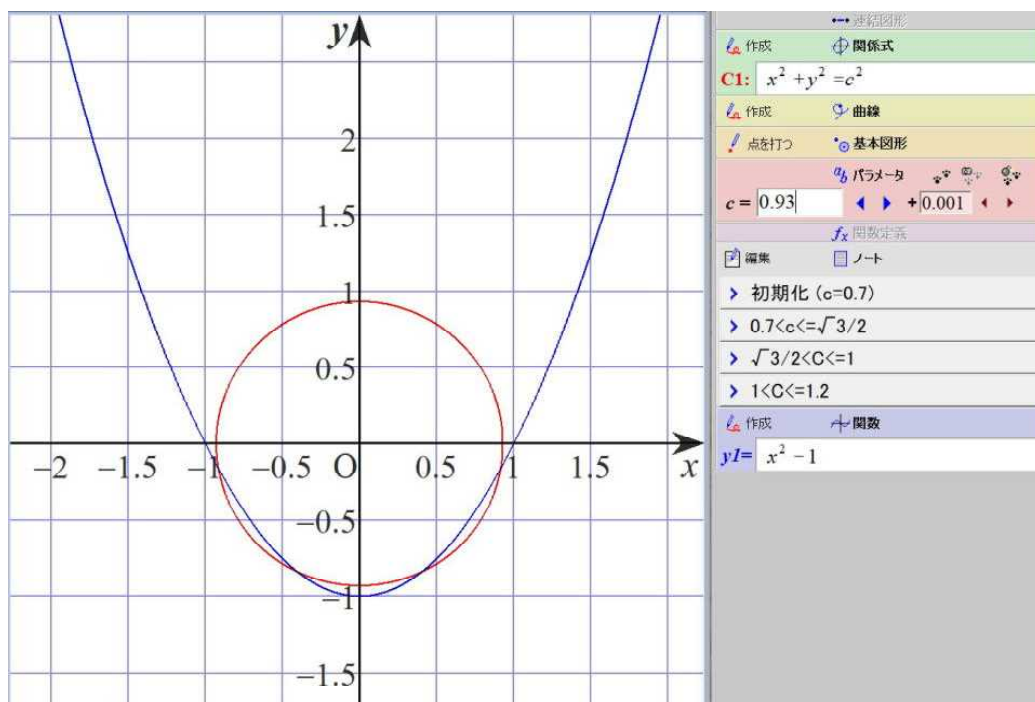
## 2 Kansei Gakuin University

(2) Experimental result (Grapes version simulation)

② When the radius  $c$  of the circle is  $\sqrt{3}/2$



③ When the radius  $c$  of the circle is 0.93





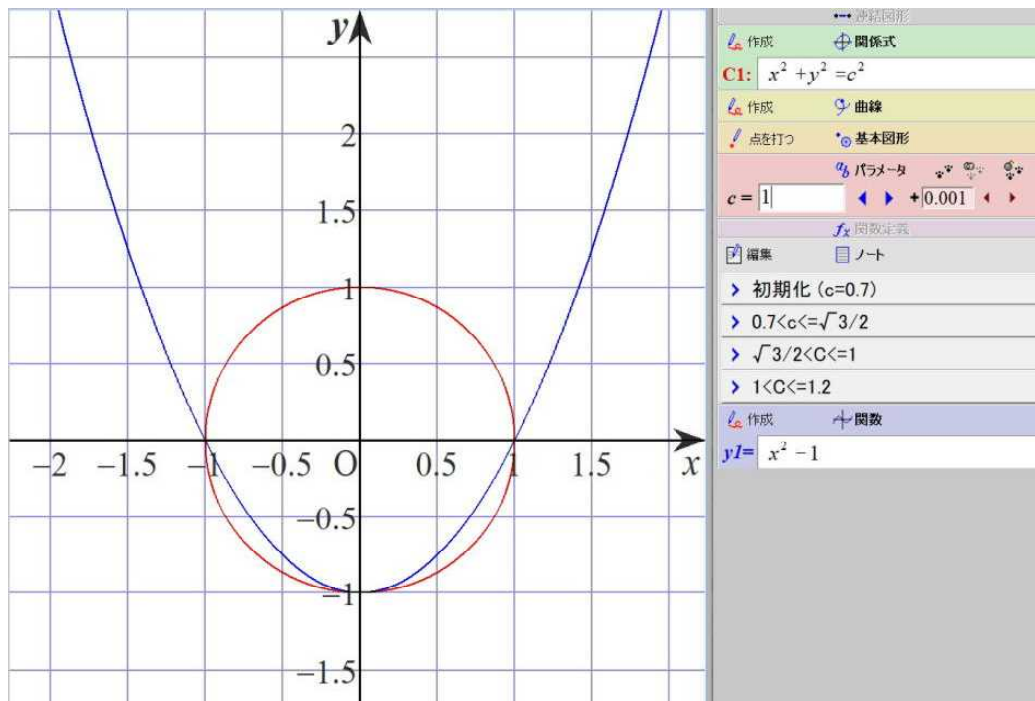
# Interesting Simulation III (Grapes)

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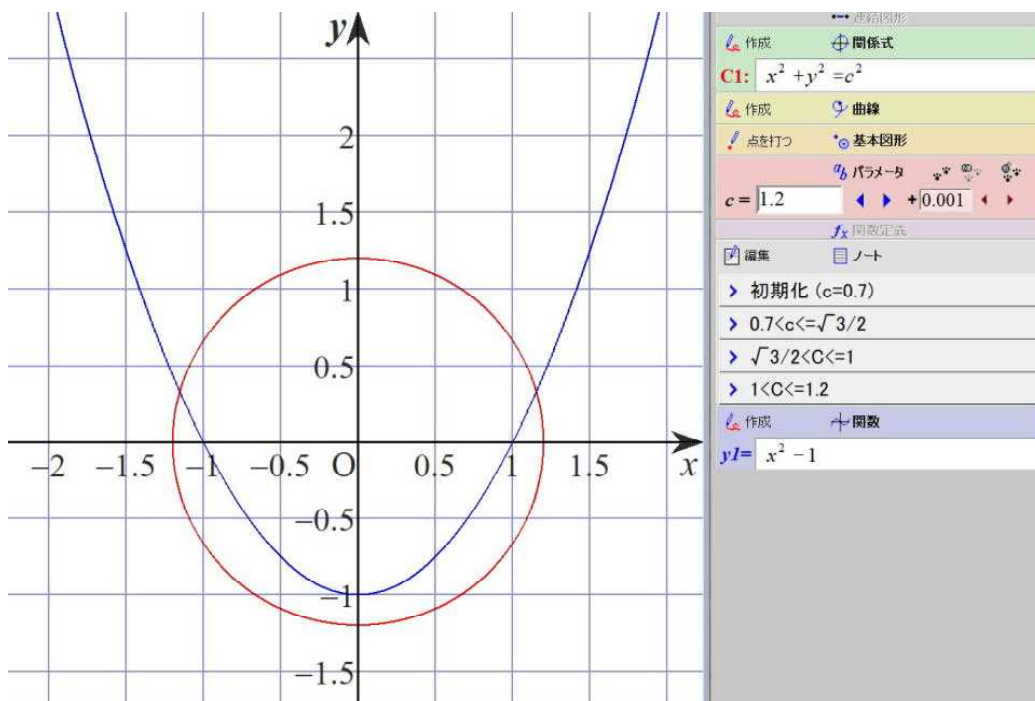
## 2 Kansei Gakuin University

(2) Experimental result (Grapes version simulation)

④ When the radius  $c$  of the circle is 1



⑤ When the radius  $c$  of the circle is 1.2





# Interesting Simulation III (Grapes)

1.22.2024  
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## 3 Shotoku Gakuen University

### (1) Entrance exam questions

For a parabola  $y=(x-p)^2-3$ , find the range of real number  $p$  such that this parabola intersects a triangle with three points  $(0,0)$ ,  $(0,-2)$ , and  $(2,0)$  as vertices.

### (2) Experimental result (Grapes version simulation)

#### 【Experiment day】

January 22 . 2024

#### 【PC used】

Lavie NX850/N

#### 【GRAPES used】

GRAPES 7.84

#### 【Script used】

Self-made file

『syoutoku.gps』

#### 【Consideration】

I varied the  $x$ -coordinate  $p$  of the vertex of the parabola from  $-2$  to  $4$  in  $0.01$  increments and observed. The parabola intersects the triangle from the time the parabola passes through the triangle's vertex  $(0,0)$  until the time it passes through the triangle's vertex  $(2,0)$ .

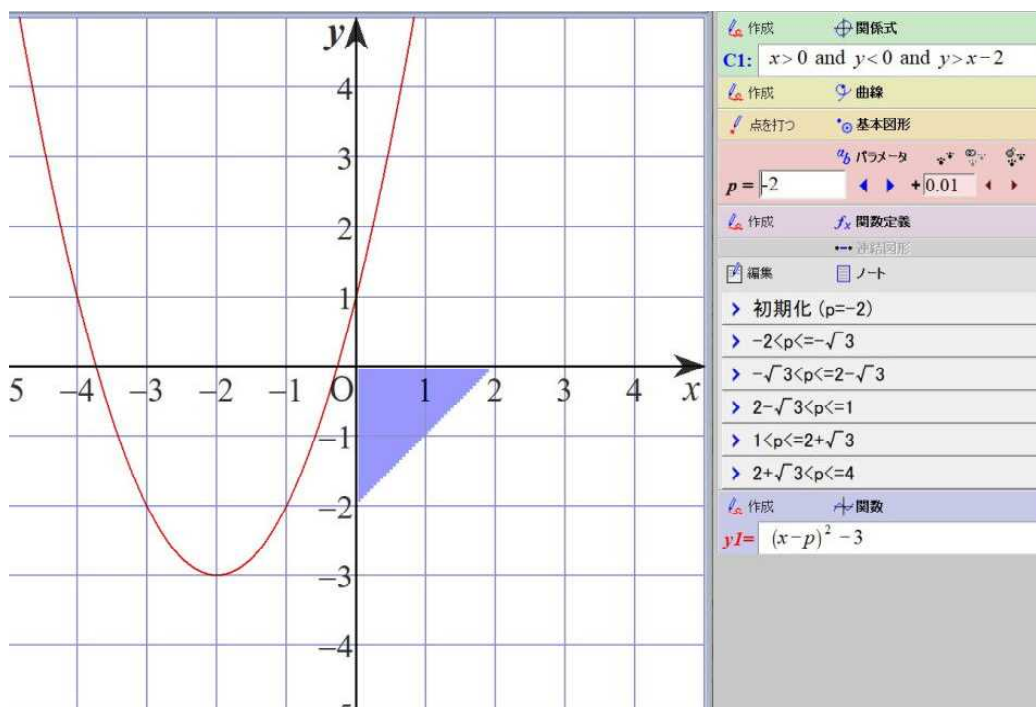
Furthermore, the parabola intersects the triangle from the time the parabola passes through the triangle's vertex  $(0,-2)$  until the time it passes through the triangle's vertex  $(2,0)$ .

When the parabola passes through  $(0,0)$ ,  $p=-\sqrt{3}$ , and when it passes through  $(2,0)$ ,  $p=2+\sqrt{3}$ .

When the parabola passes through  $(0,-2)$ ,  $p=1$ , and when it passes through  $(2,0)$ ,  $p=2+\sqrt{3}$ .

Therefore,  $-\sqrt{3} \leq p \leq 2-\sqrt{3}$ 、 $1 \leq p \leq 2+\sqrt{3}$

### ① When the value of $p$ is $-2$



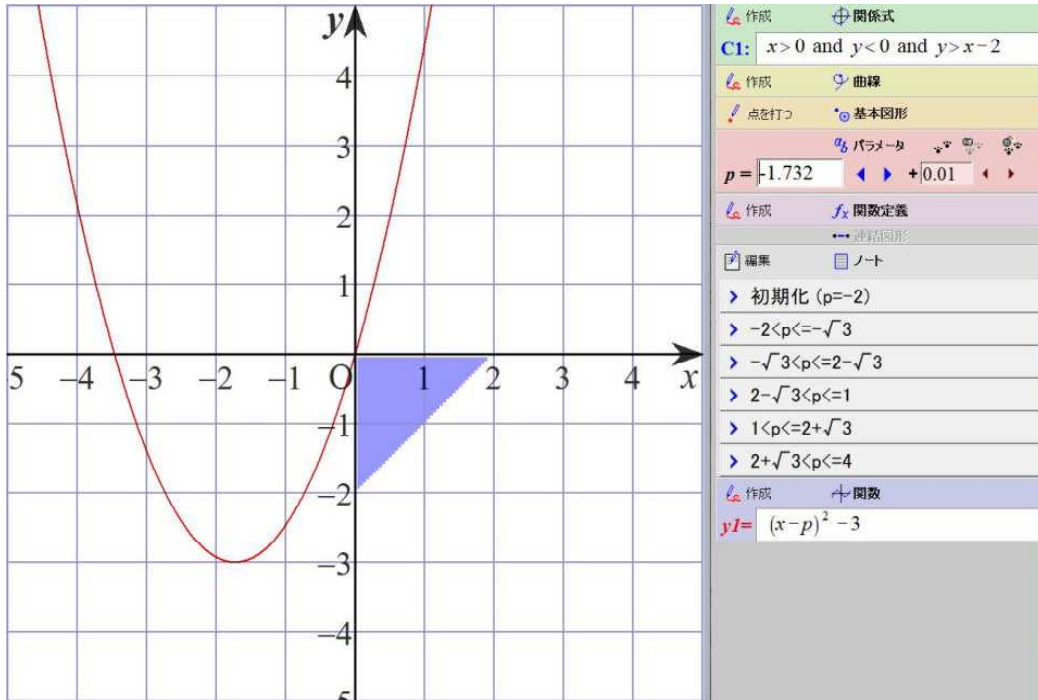
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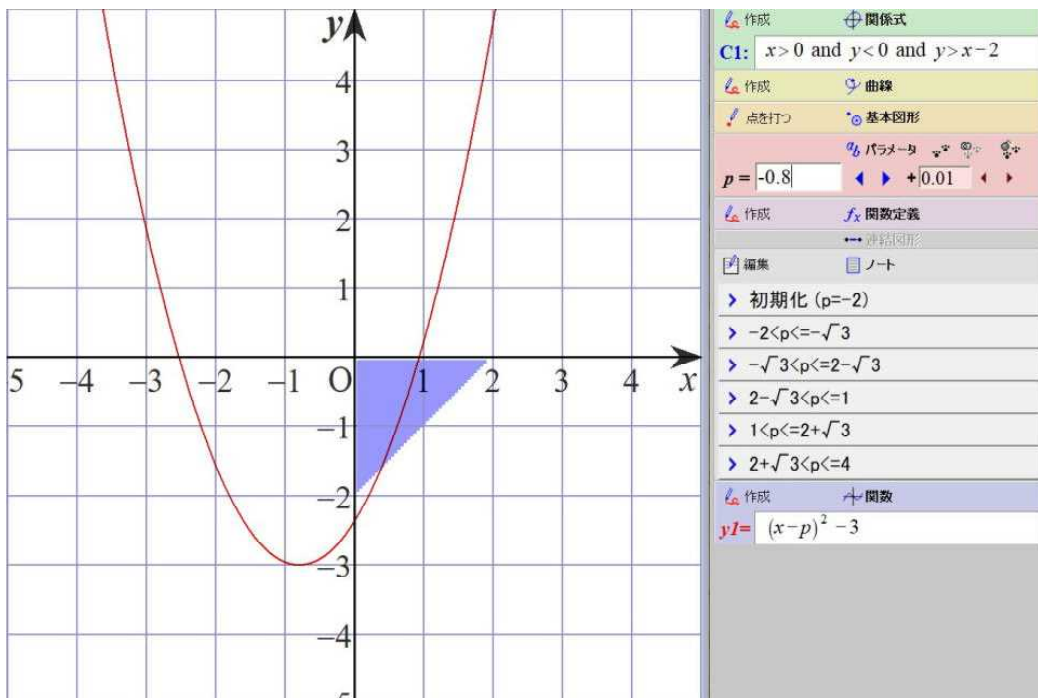
## 3 Shotoku Gakuen University

(2) Experimental result (Grapes version simulation)

② When the value of  $p$  is  $-\sqrt{3}$



③ When the value of  $p$  is  $-0.8$



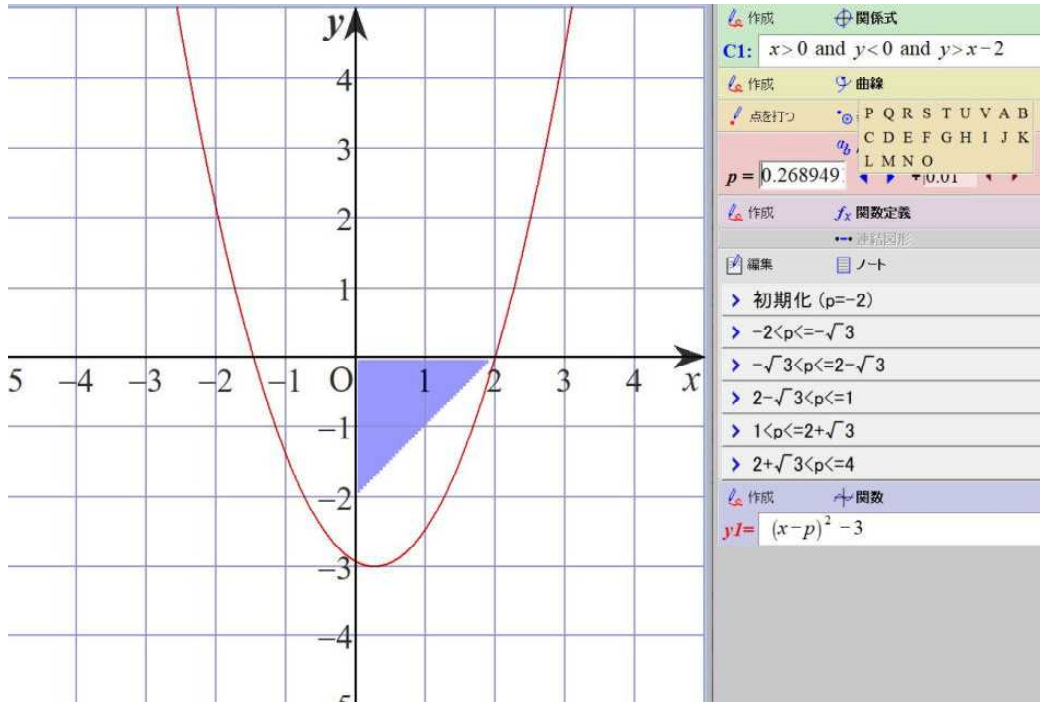
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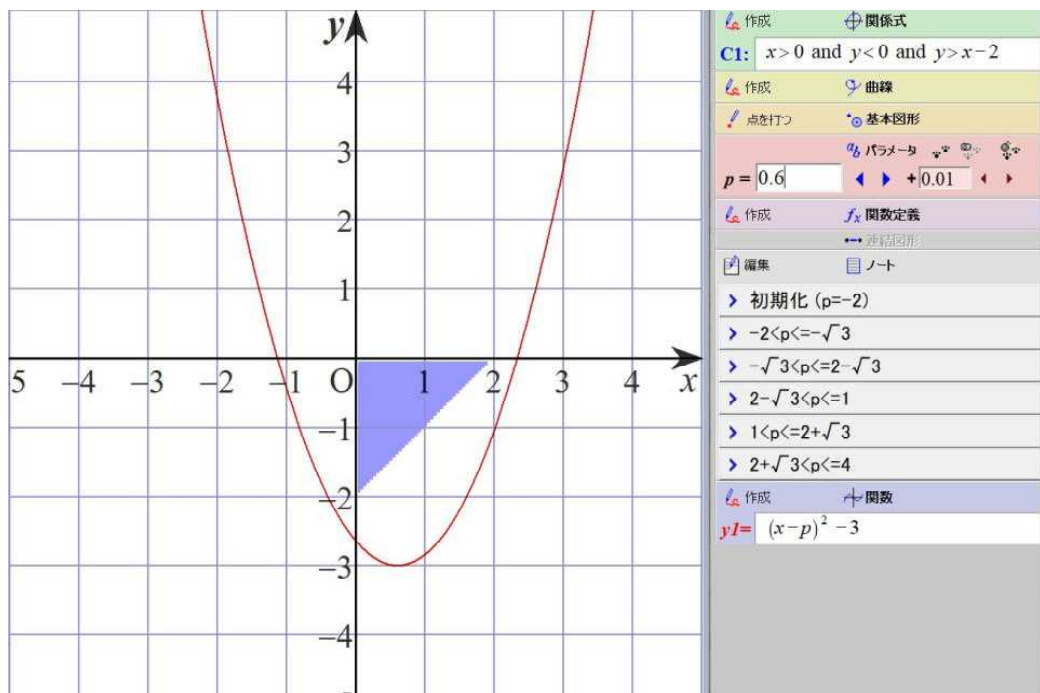
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(2) Experimental result (Grapes version simulation)

④ When the value of  $p$  is  $2 - \sqrt{3}$



⑤ When the value of  $p$  is 0.6





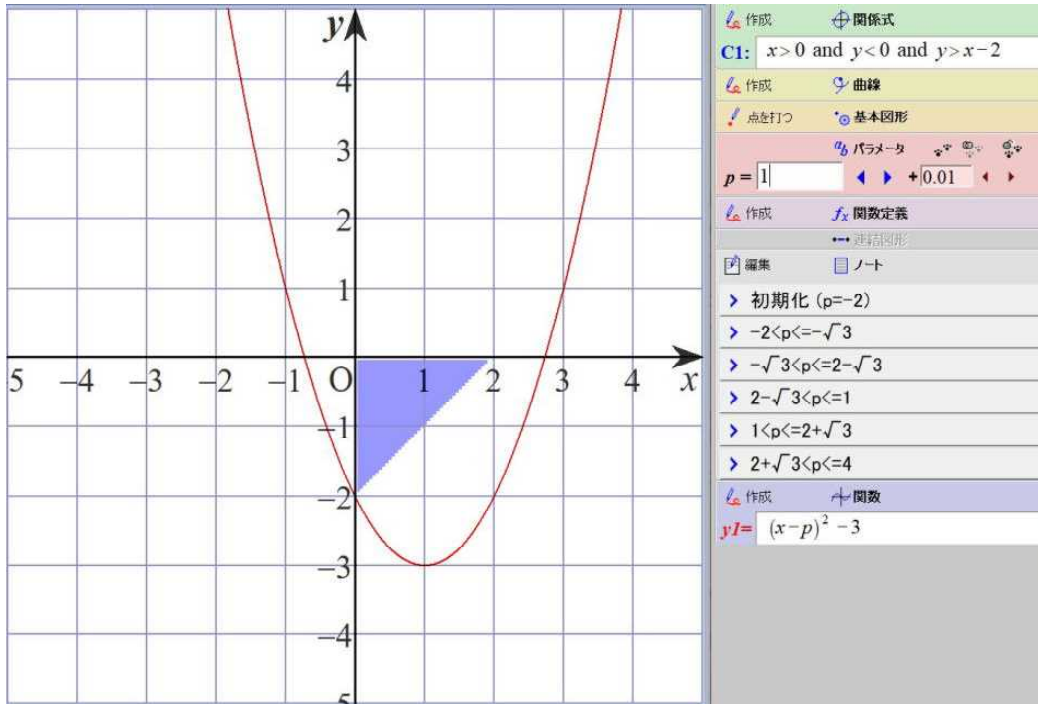
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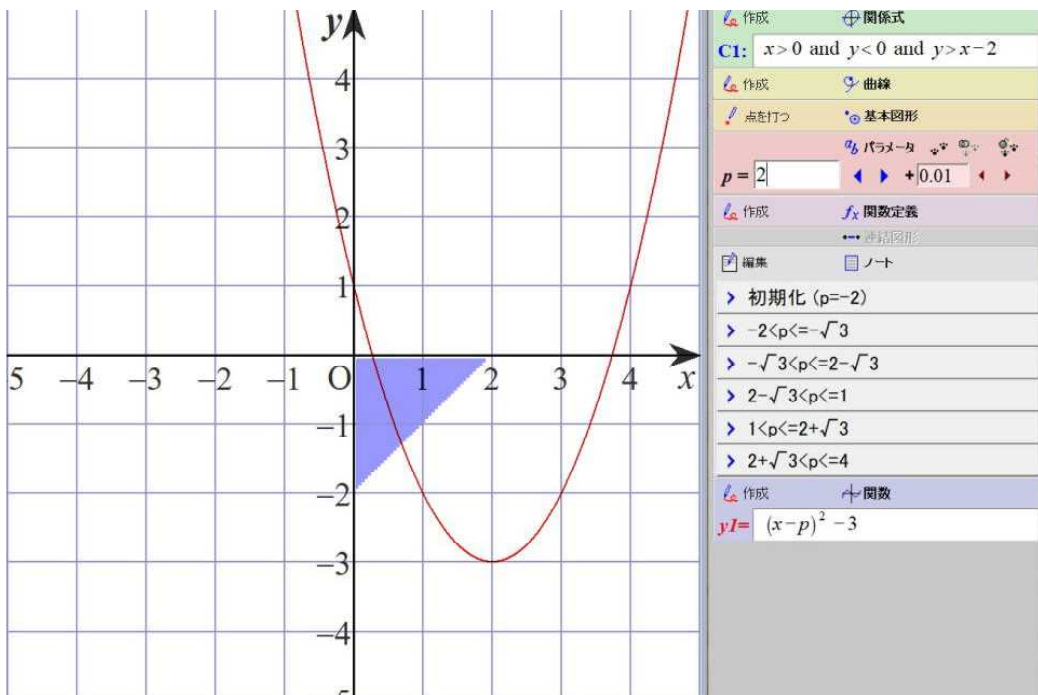
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(2) Experimental result (Grapes version simulation)

⑥ When the value of p is 1



⑦ When the value of p is 2



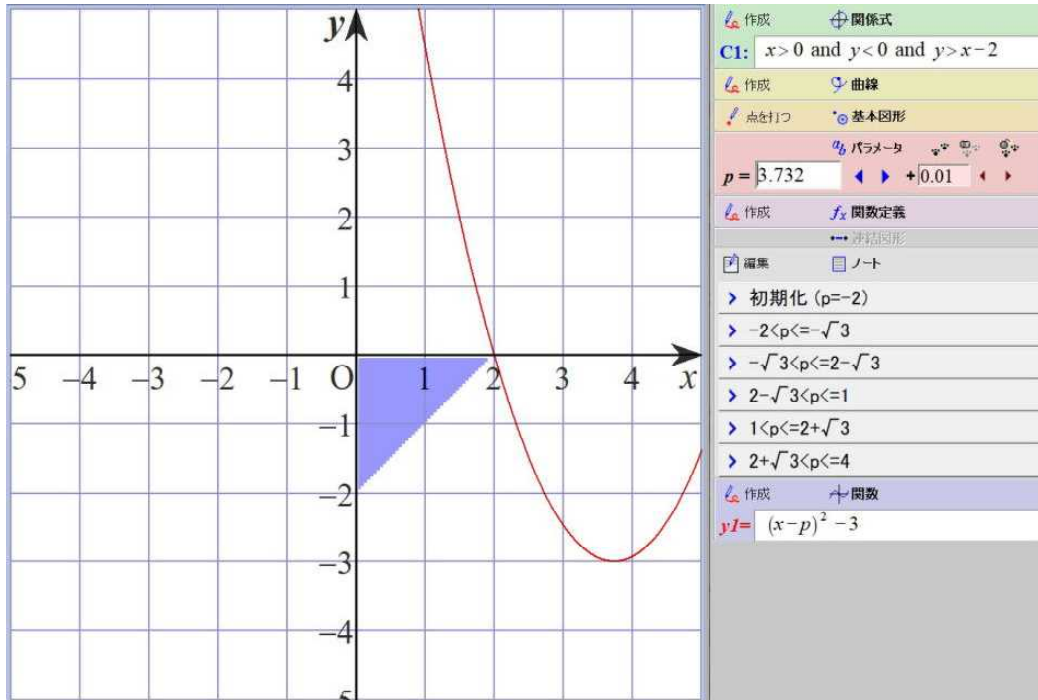
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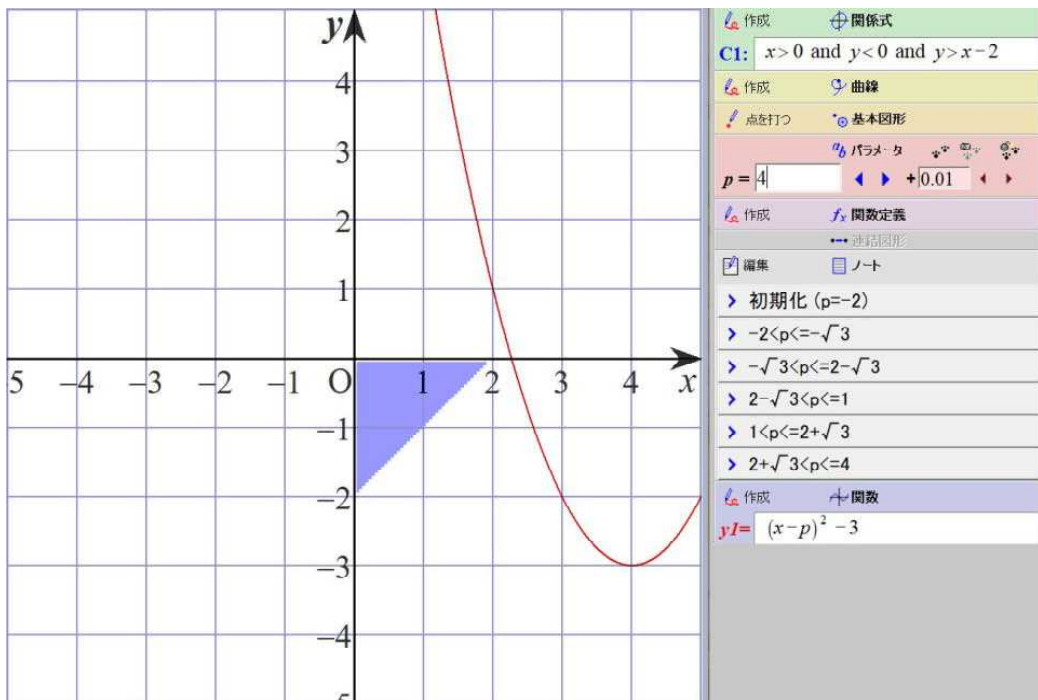
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(2) Experimental result (Grapes version simulation)

⑧ When the value of  $p$  is  $2 + \sqrt{3}$



⑨ When the value of  $p$  is 4



# Interesting Simulation III (Grapes)

1.23.2024  
Sohun

## 4 Soka University

### (1) Entrance exam questions

When ③, which passes through the two intersections of the two circles ①:  $x^2 + y^2 = 2$  and ②:  $(x-1)^2 + (y+1)^2 = 1$ , touches the straight line  $y=x$ , find the center and radius of that circle.

### (2) Experimental result (Grapes version simulation)

#### 【Experiment day】

January 23, 2024

#### 【PC used】

Lavie NX850/N

#### 【GRAPES used】

GRAPES 7.84

#### 【Script used】

Self-made file  
『souka.gps』

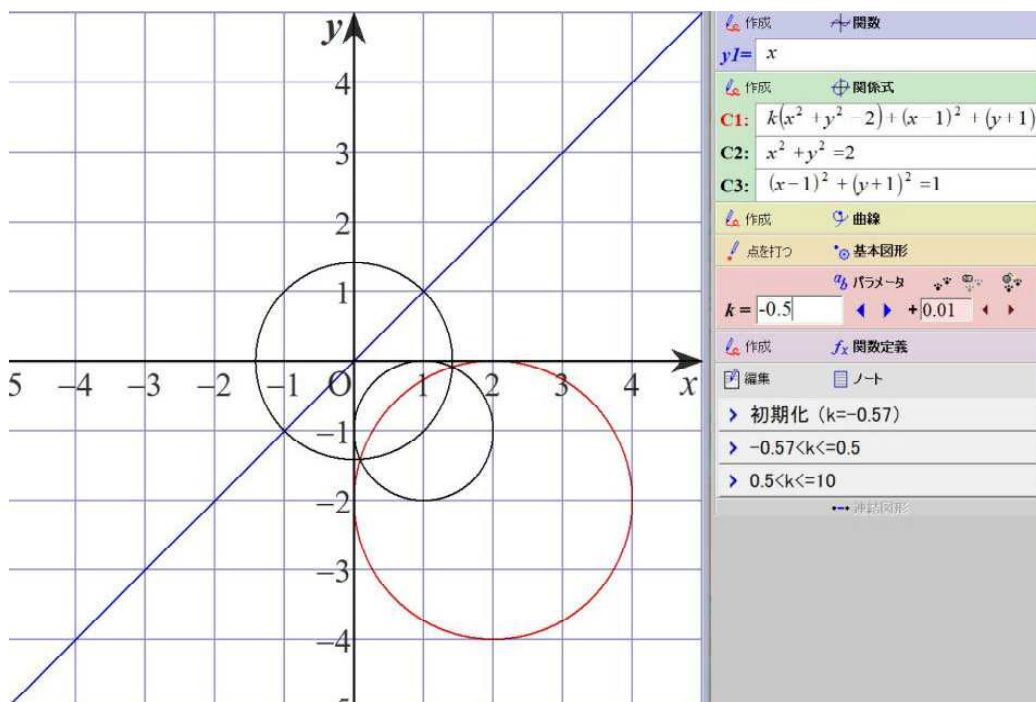
#### 【Consideration】

A circle passing through the intersection of two circles ① and ② can be expressed as  
①:  $k(x^2 + y^2 - 2) + (x-1)^2 + (y+1)^2 - 1 = 0$ . I varied the value of  $k$  in this circle equation from -0.57 to 10 and from -0.57 to -30, and observed. When  $-0.57 < k \leq 0$ , circle ③ is away from the straight line  $y=x$ . When  $0 < k \leq 0.5$ , circle ③ approaches the straight line  $y=x$  and touches the straight line  $y=x$ . When  $0.5 < k$ , circle ③ intersects the straight line  $y=x$  at two points. when  $-0.57 > k > -1$ , circle ③ is away from the straight line  $y=x$ . When  $k = -1$ , ③ is not a circle but a straight line. When  $-1 > k$ , circle ③ intersects the straight line  $y=x$  at two points.

Therefore, calculate and find the value of  $k$  when circle ③ touches the straight line  $y=x$ .

$$k = \frac{1}{2} \quad \text{center} \left( \frac{2}{3}, -\frac{2}{3} \right), \quad \text{radius} \frac{2\sqrt{2}}{3}$$

#### ① When the value of $k$ is -0.5





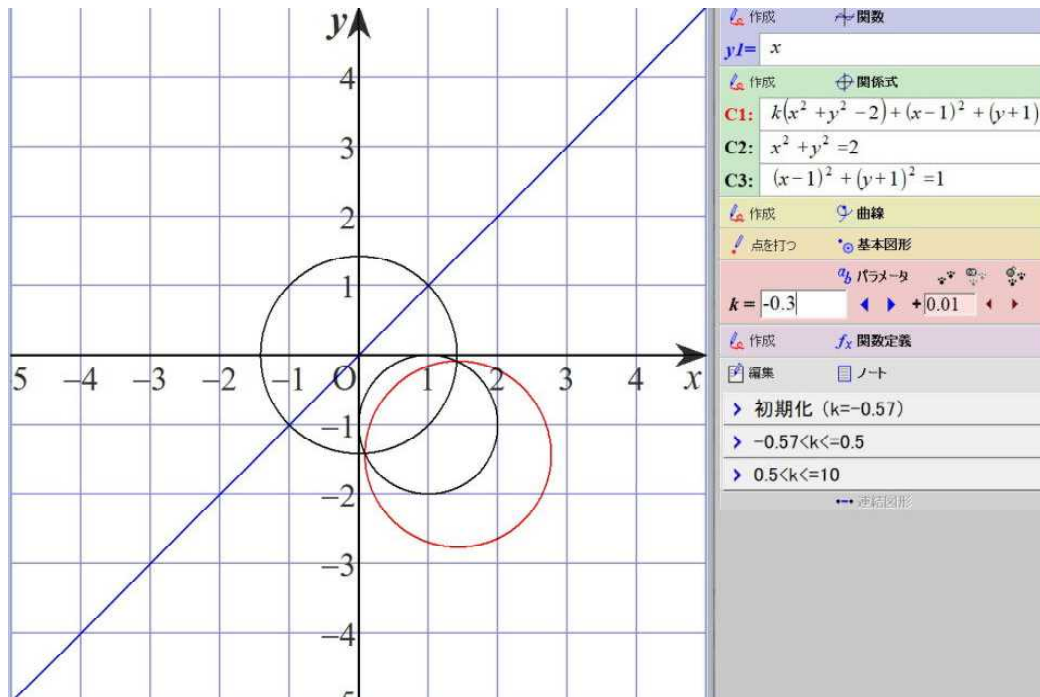
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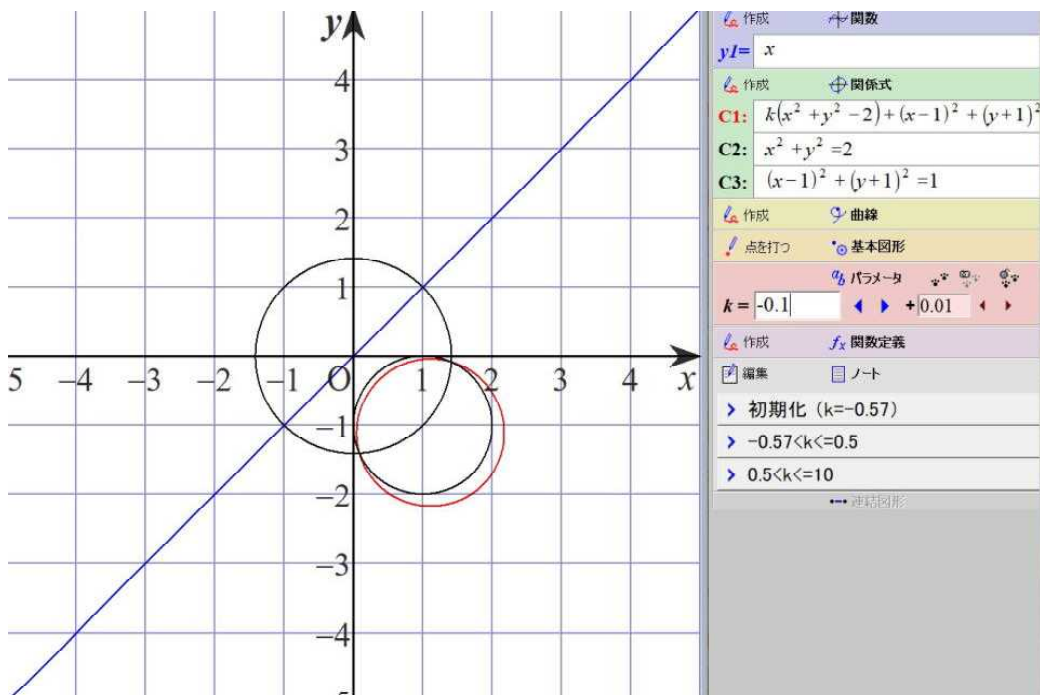
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(2) Experimental result (Grapes version simulation)

② When the value of k is -0.3



③ When the value of k is -0.1



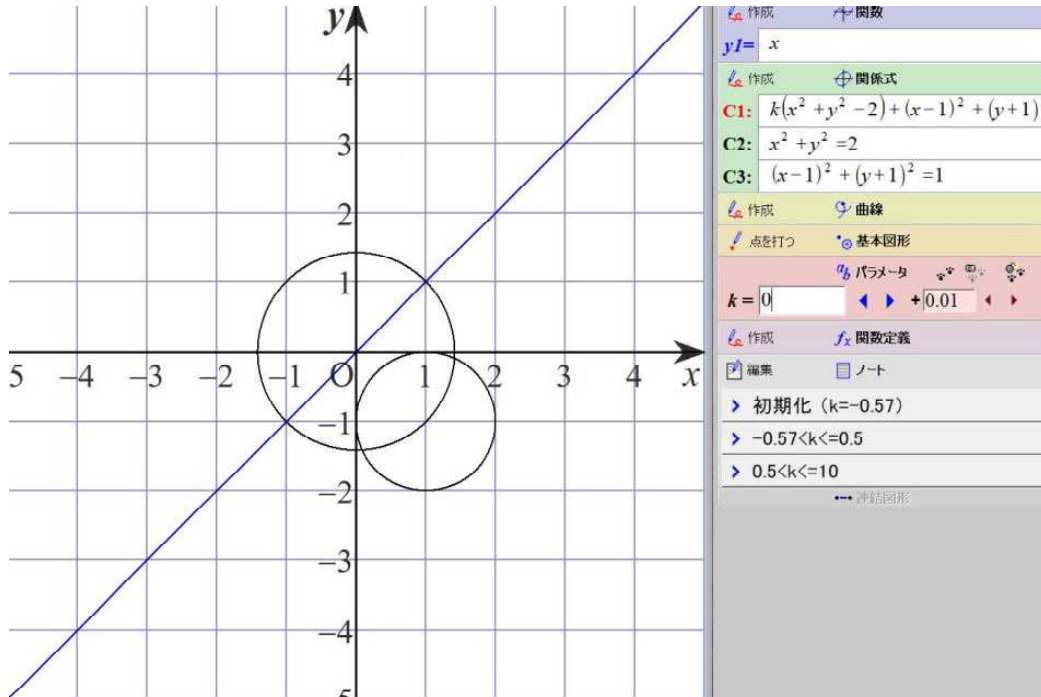
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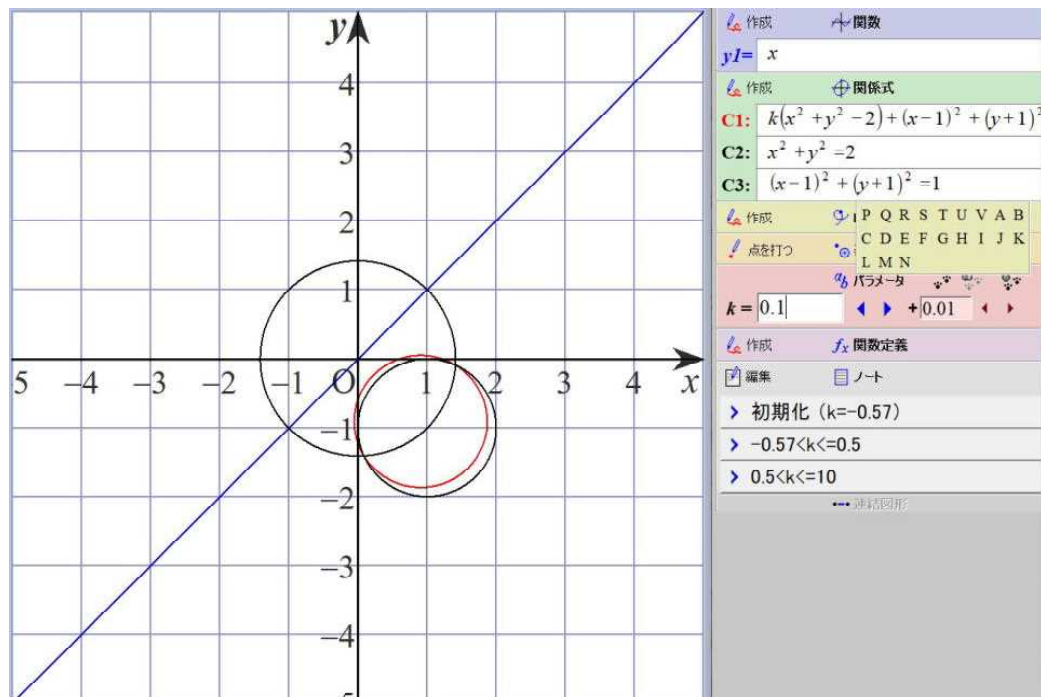
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(2) Experimental result (Grapes version simulation)

④ When the value of k is 0



⑤ When the value of k is 0.1



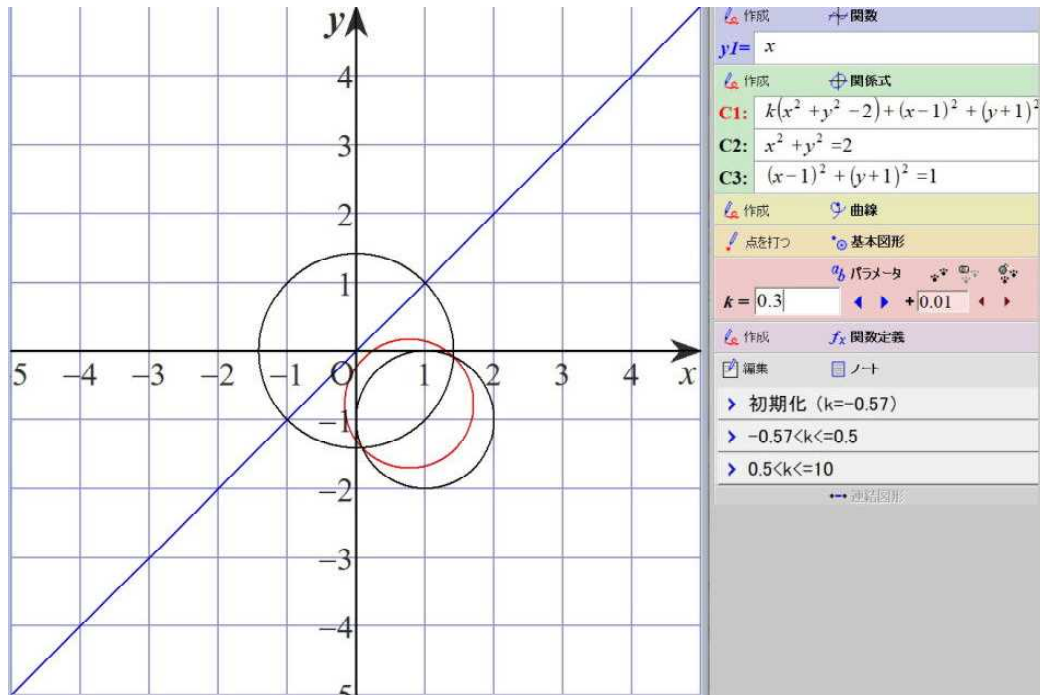
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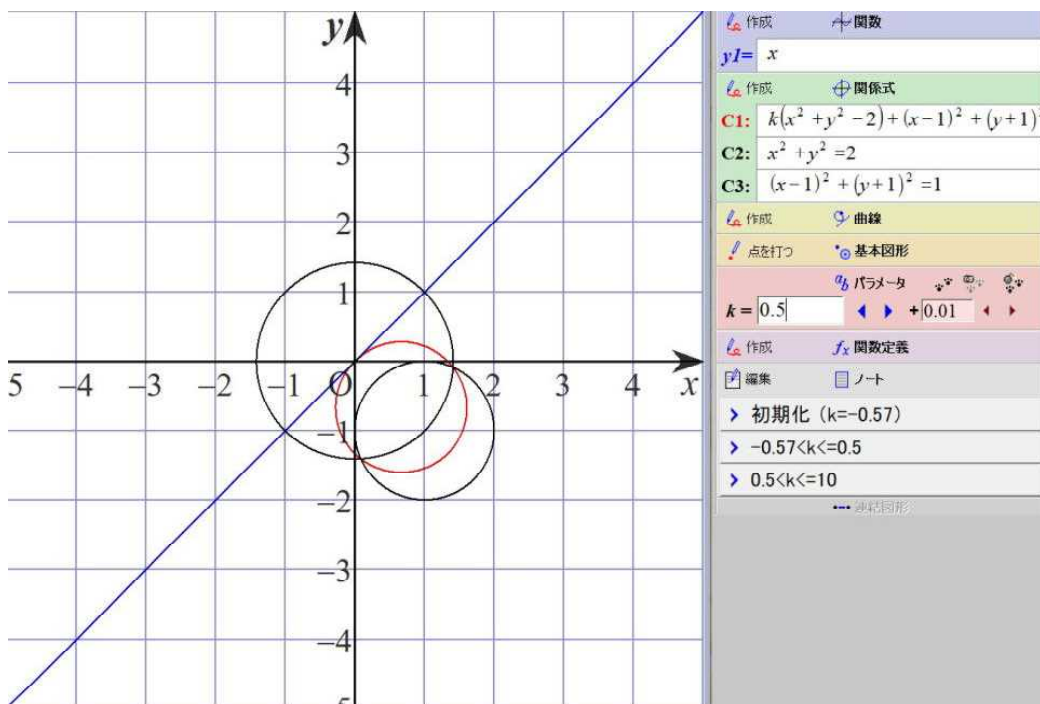
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(2) Experimental result (Grapes version simulation)

⑥ When the value of k is 0.3



⑦ When the value of k is 0.5





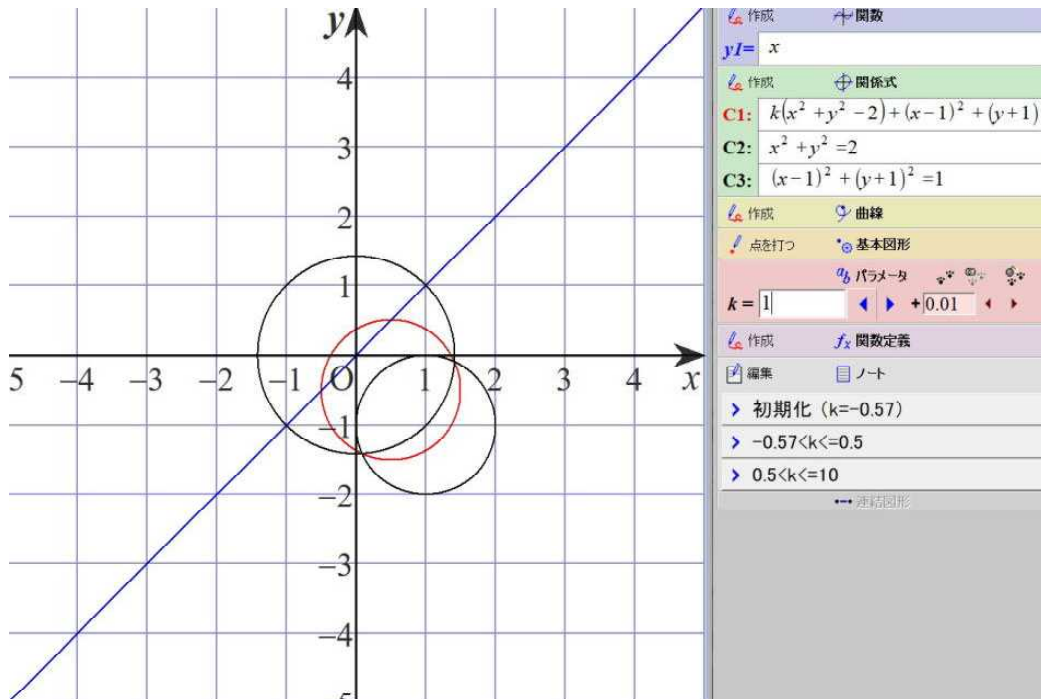
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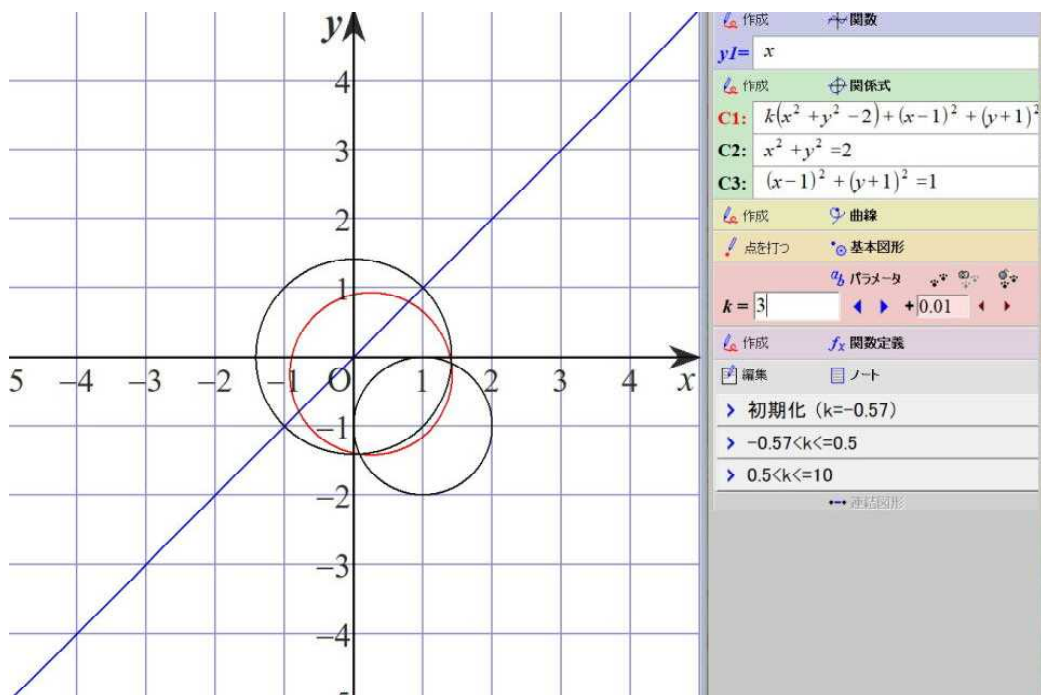
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(2) Experimental result (Grapes version simulation)

⑧ When the value of k is 1



⑨ When the value of k is 3



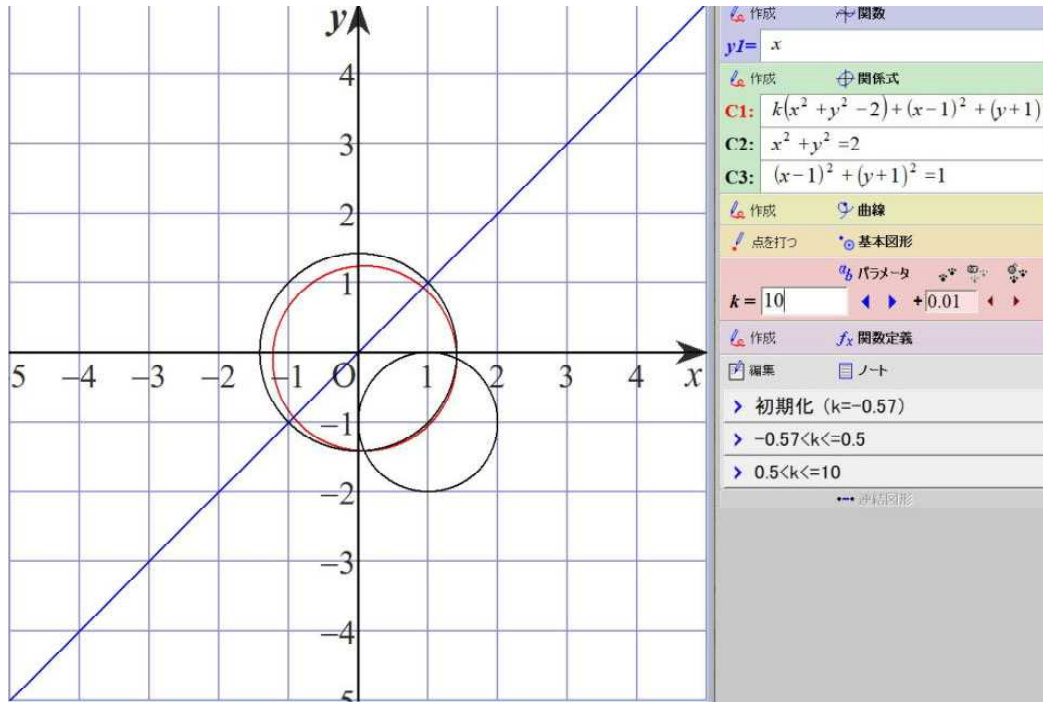
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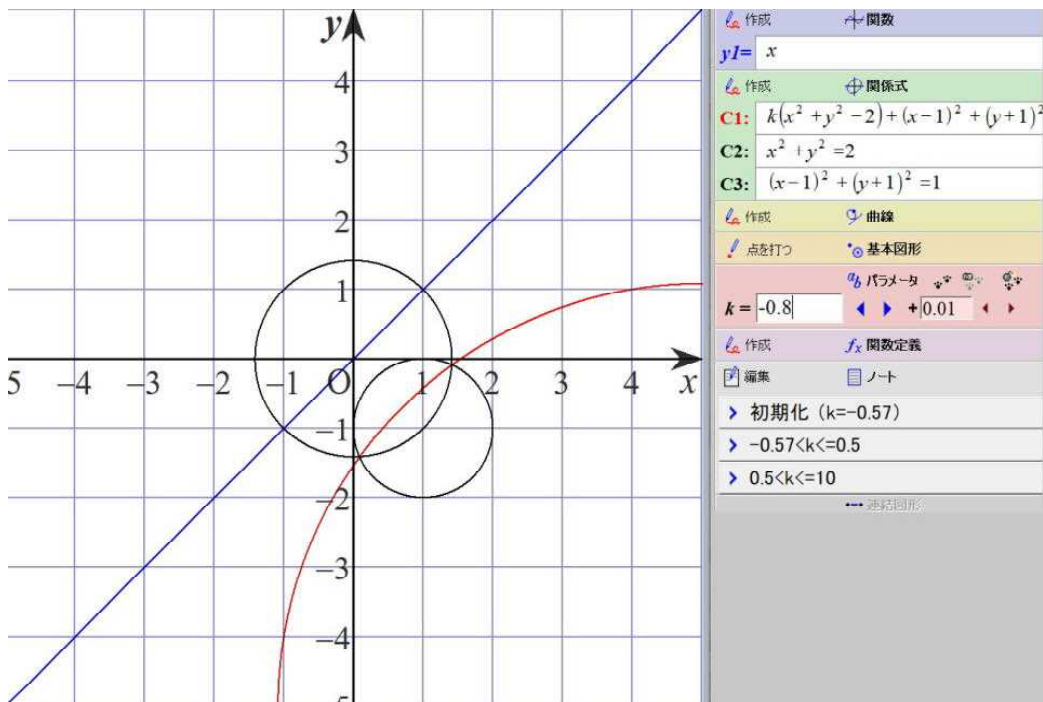
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(2) Experimental result (Grapes version simulation)

⑩ When the value of k is 10



⑪ When the value of k is -0.8



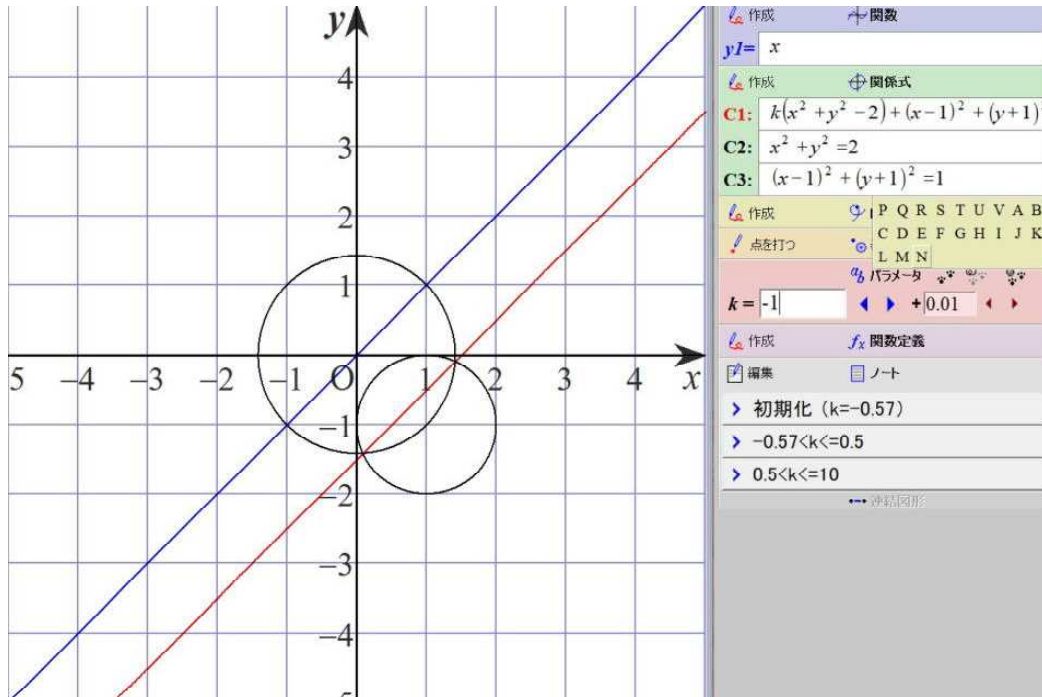
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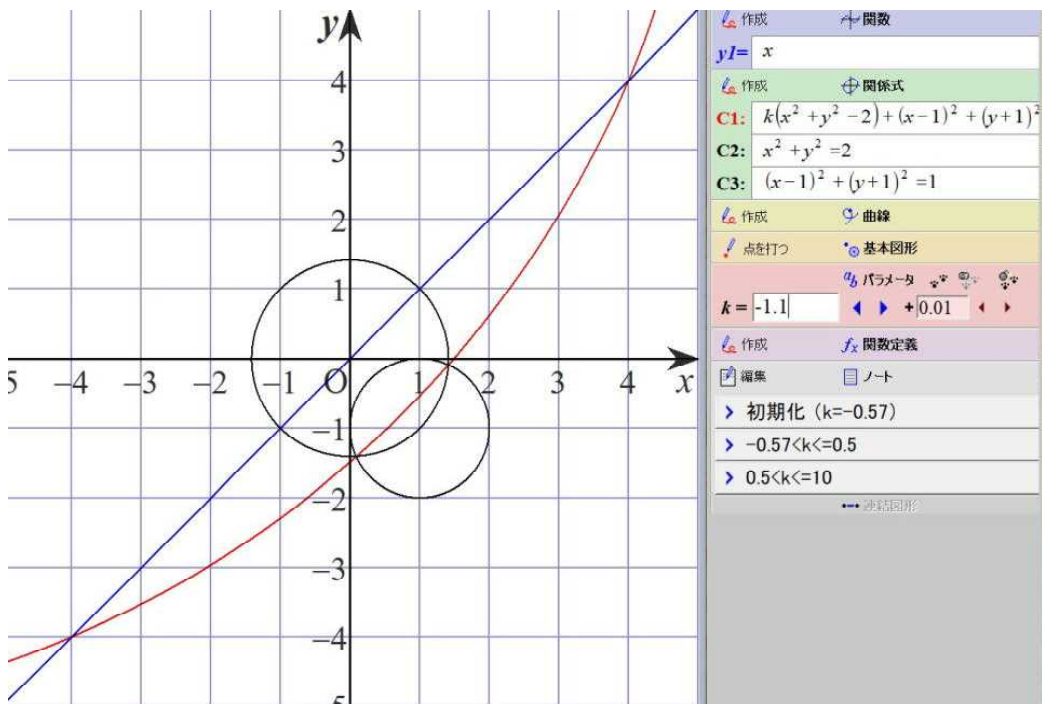
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(2) Experimental result (Grapes version simulation)

⑫ When the value of k is -1



⑬ When the value of k is -1.1



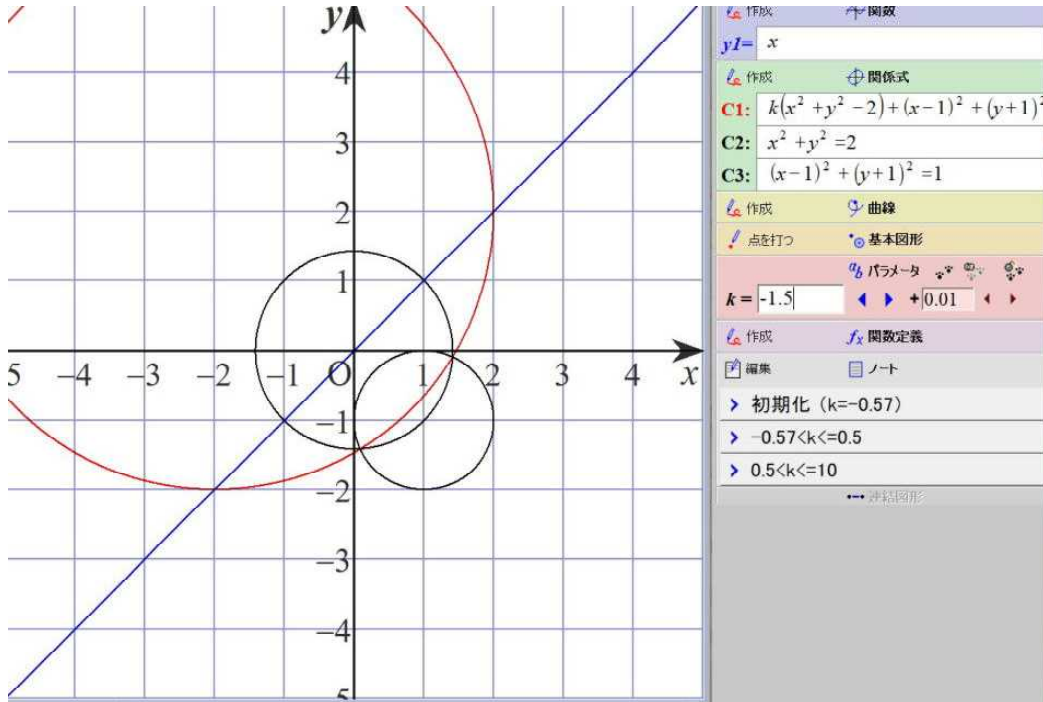
# Interesting Simulation III (Grapes)

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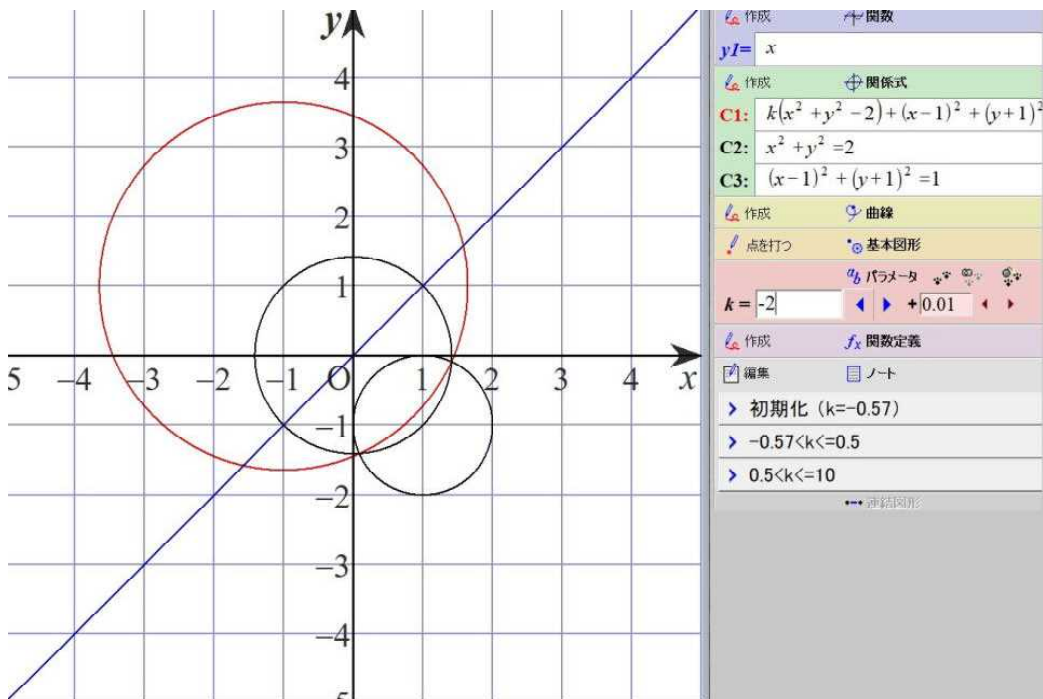
## 4 Soka University

(2) Experimental result (Grapes version simulation)

⑭ When the value of k is -1.5



⑮ When the value of k is -2





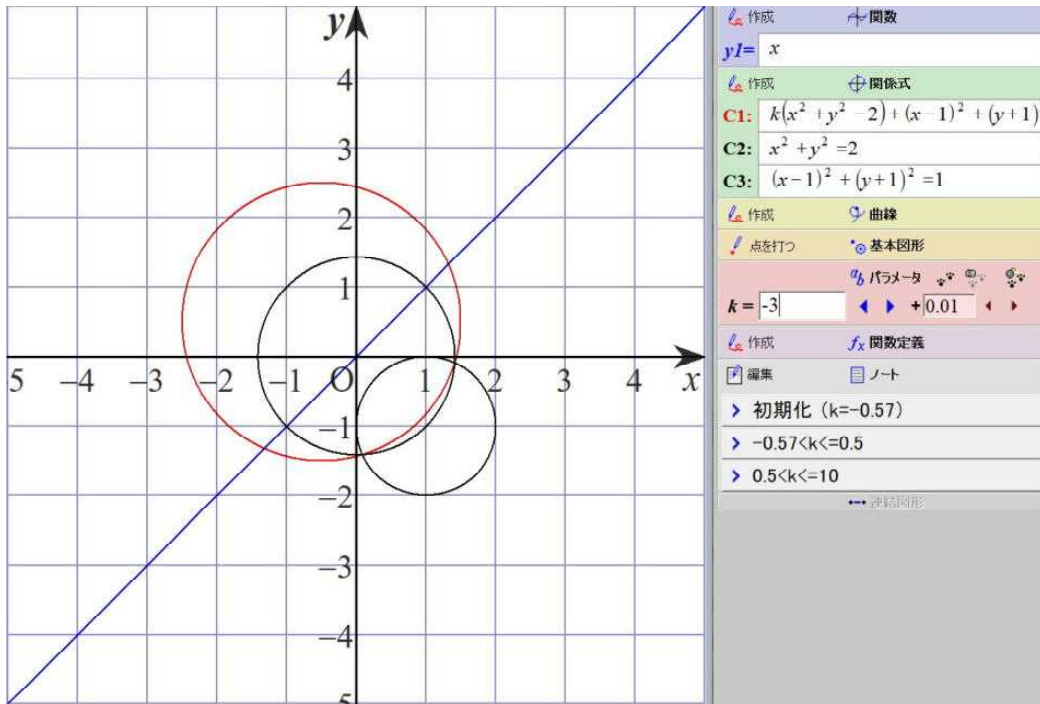
# Interesting Simulation III (Grapes)

1.23.2024  
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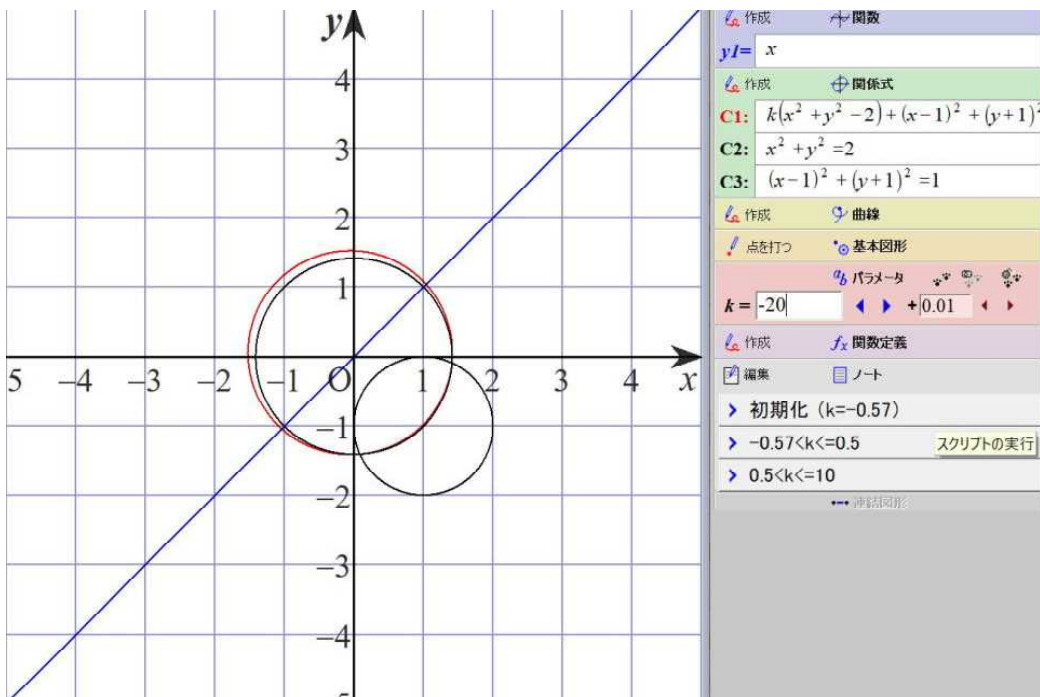
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(2) Experimental result (Grapes version simulation)

⑩ When the value of k is -3



⑪ When the value of k is -20



# Interesting Simulation III (Grapes)

1.24.2024  
Sohun

## 5 Fukuoka University

### (1) Entrance exam questions

When  $k$  is a constant satisfying  $k \neq -\sqrt{2}$ , ①:  $x^2 + y^2 - 1 + k(x - y - \sqrt{2}) = 0$  represents a circle passing through fixed point  $A$  regardless of the value of  $k$ . At this time, find the coordinates of fixed point  $A$ .

Also, when circle ① and circle ②:  $(x-1)^2 + (y-1)^2 = 9$  have one common point and  $k > 0$ , find the value of  $k$ .

### (2) Experimental result (Grapes version simulation) $k \neq -\sqrt{2}$

#### 【Experiment day】

January 24 . 2024

#### 【PC used】

Lavie NX850/N

#### 【GRAPES used】

GRAPES 7.84

#### 【Script used】

Self-made file

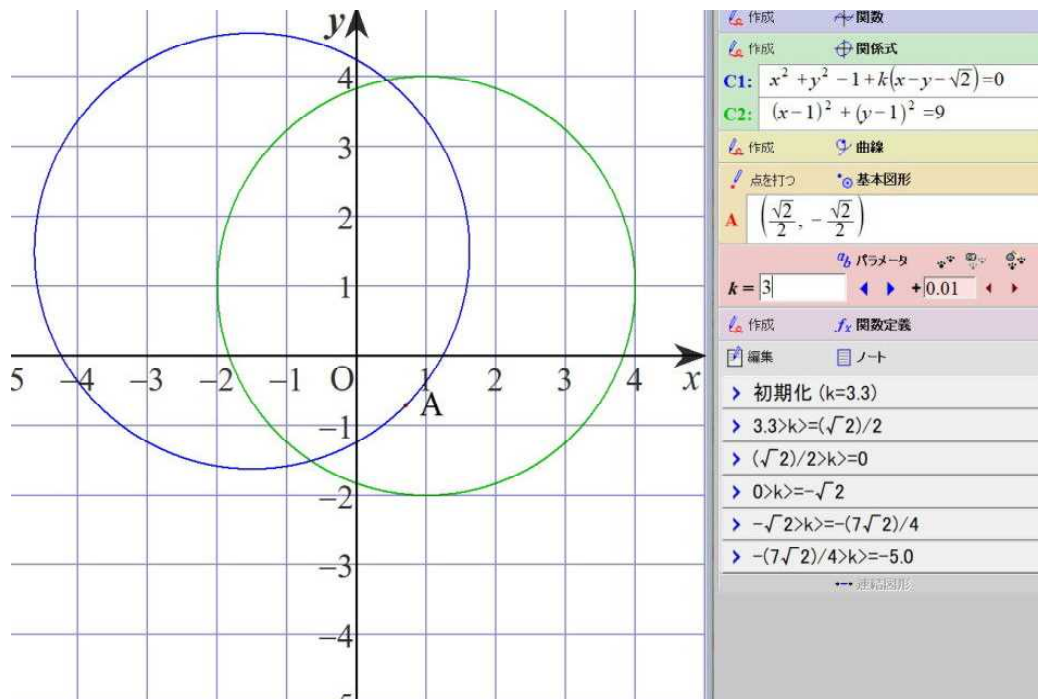
『fukuoka.gps』

#### 【Consideration】

I varied the value of  $k$  in the circle ① equation from 3.3 to -5 in 0.01 increments, and observed.

When  $k > \sqrt{2}/2$ , circle ① and circle ② intersect at two points. When  $k = \sqrt{2}/2$ , circle ① and circle ② have only one common point. When  $\sqrt{2}/2 > k > -7\sqrt{2}/4$ , circle ① is inside circle ②. When  $k = -7\sqrt{2}/4$ , circle ① and circle ② have only one common point. When  $k < -7\sqrt{2}/4$ , circle ① and circle ② intersect at two points. By the way, when  $k = -\sqrt{2}$ , the circle ① disappears. Find the coordinates of fixed point  $A(\sqrt{2}/2, -\sqrt{2}/2)$  by solving simultaneous equations. Calculate and found  $k = \sqrt{2}/2$  ( $k > 0$ ) from the relationship between the radius of the two circles and the distance between their centers.

#### ① When the value of $k$ is 3



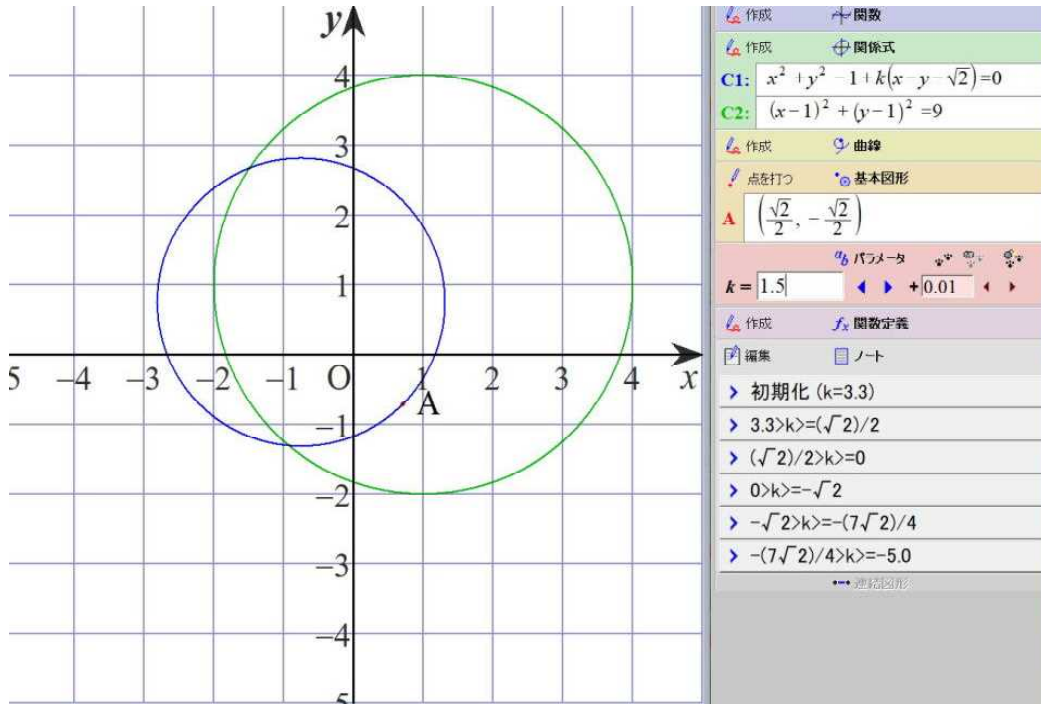
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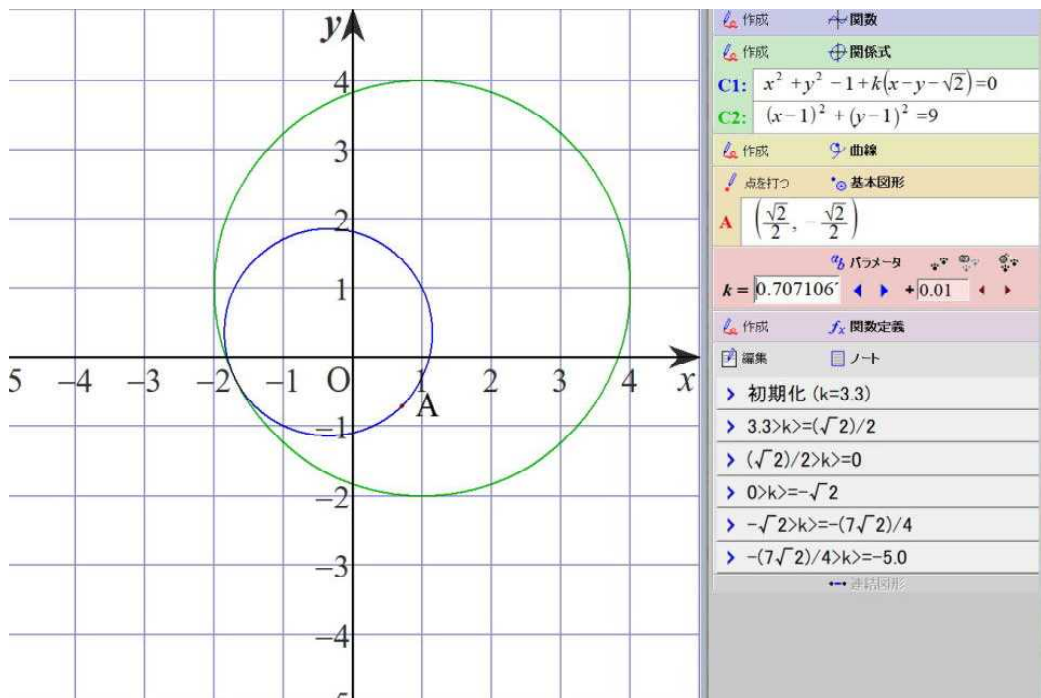
## 5 Fukuoka University

(2) Experimental result (Grapes version simulation)

② When the value of k is 1.5



③ When the value of k is  $\frac{\sqrt{2}}{2}$



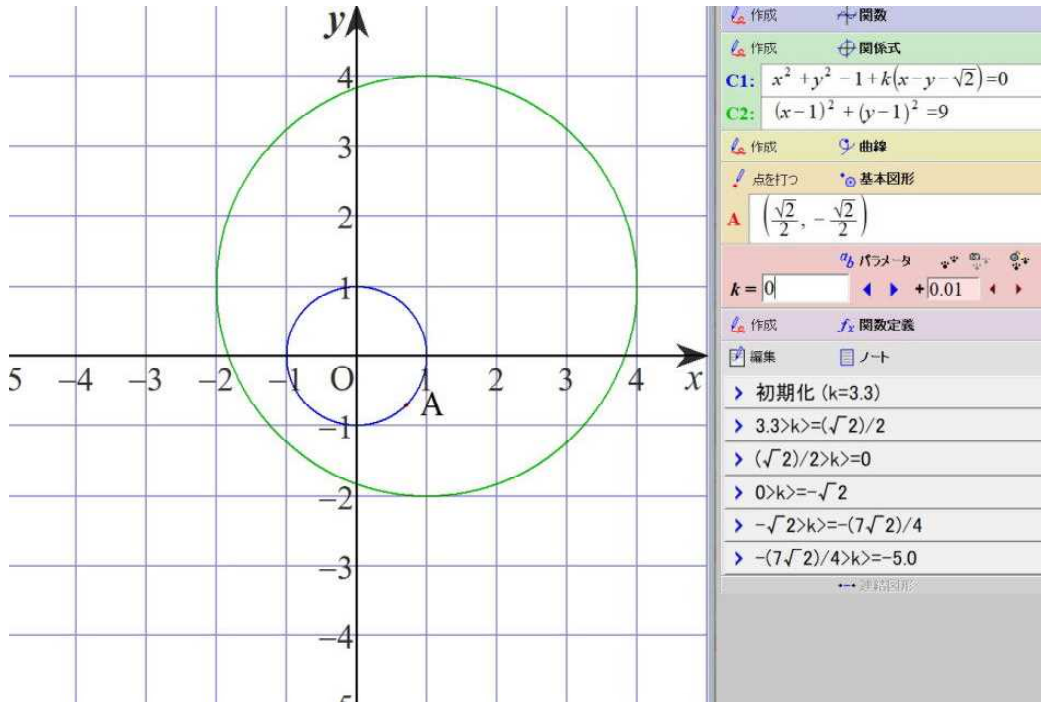
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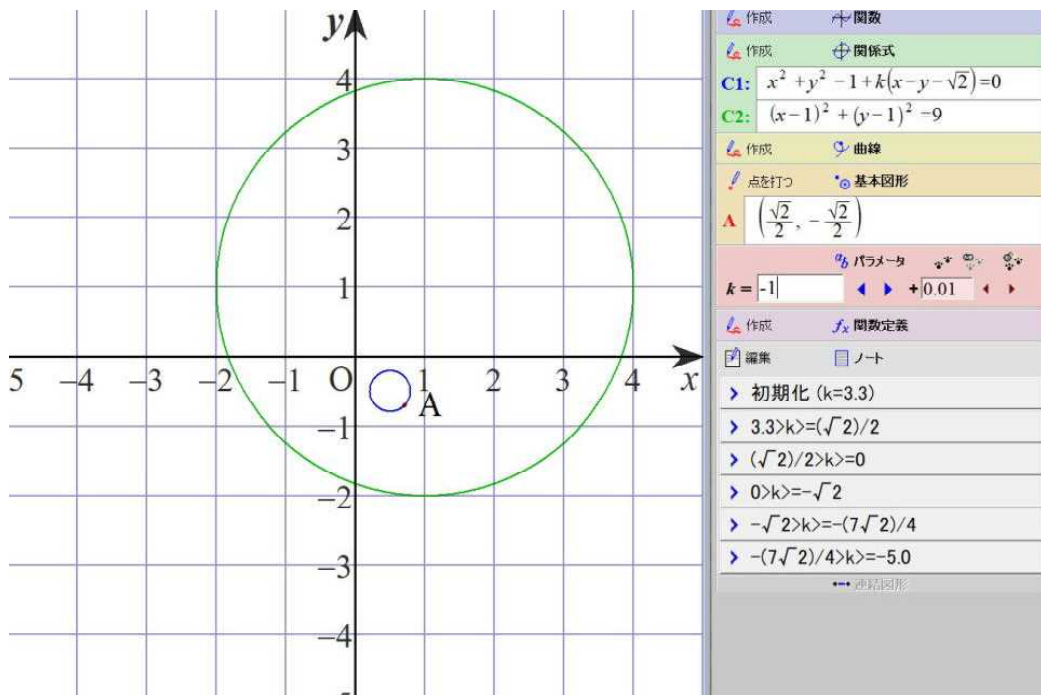
## 5 Fukuoka University

(2) Experimental result (Grapes version simulation)

④ When the value of k is 0



⑤ When the value of k is -1





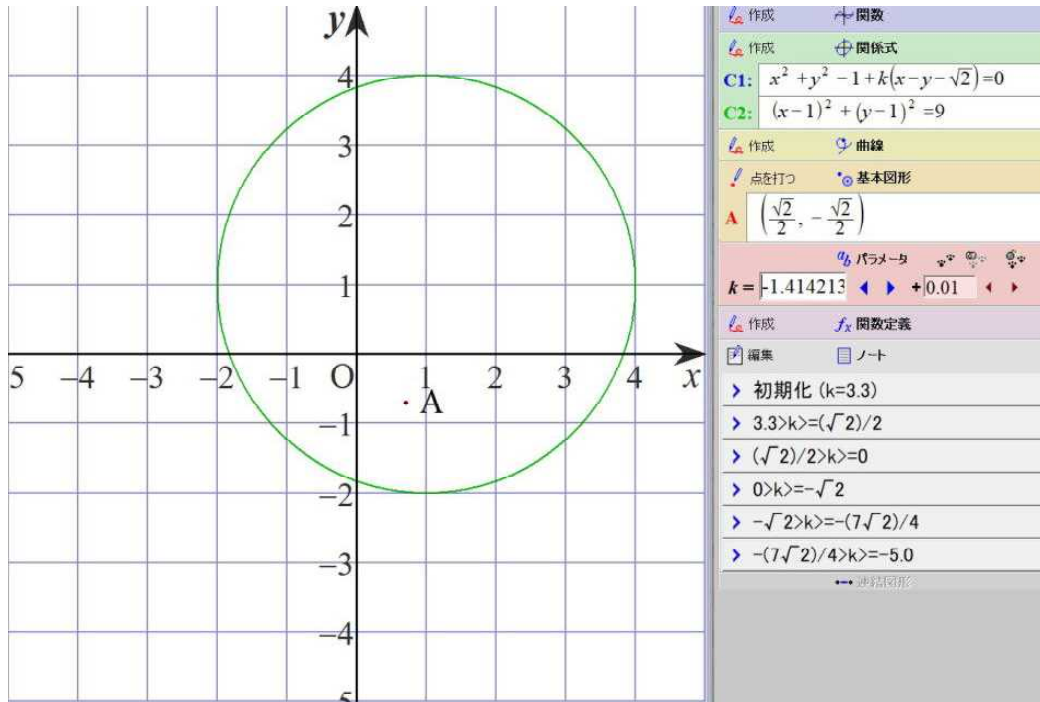
# Interesting Simulation III (Grapes)

1.24.2024  
Sohun

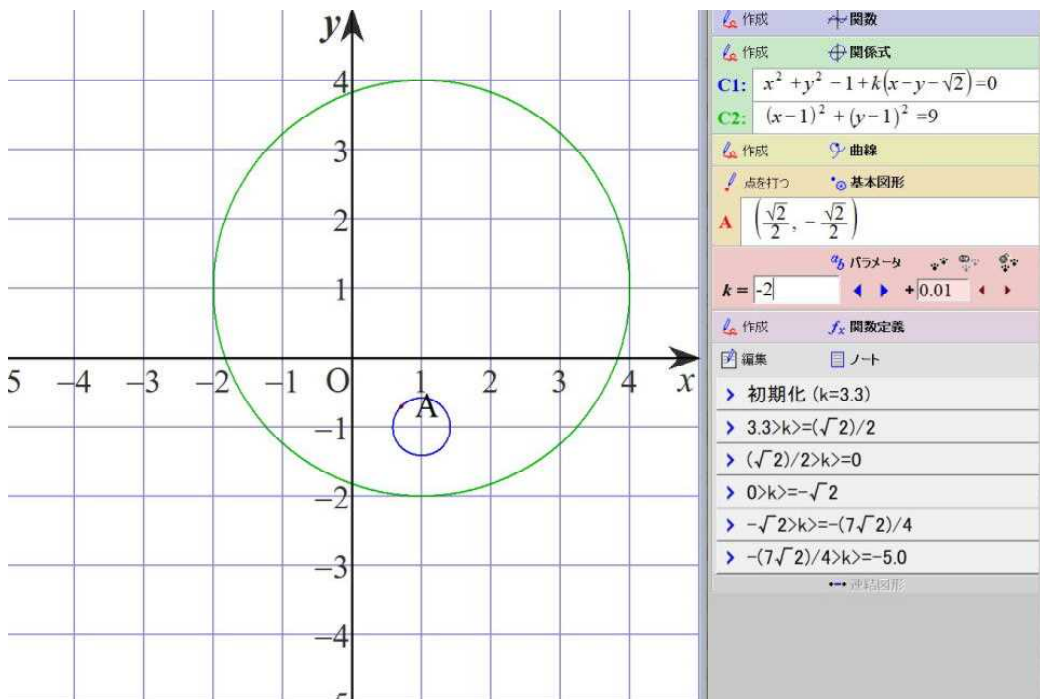
## 5 Fukuoka University

(2) Experimental result (Grapes version simulation)

⑥ When the value of  $k$  is  $-\sqrt{2}$



⑦ When the value of  $k$  is  $-2$



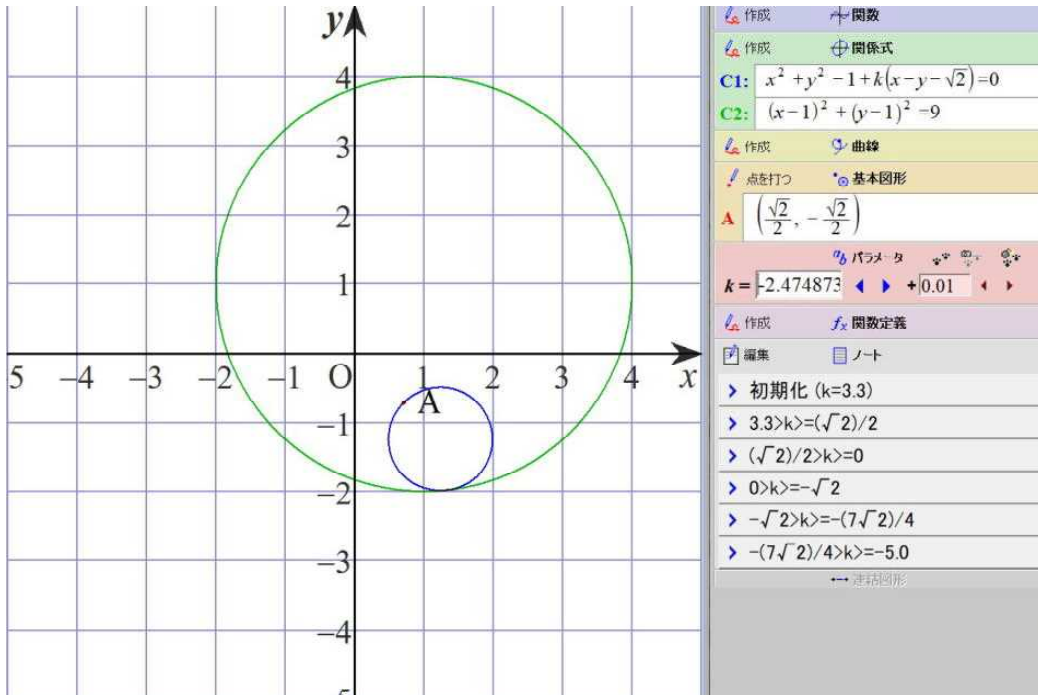
# Interesting Simulation III (Grapes)

1.24.2024  
Sohun

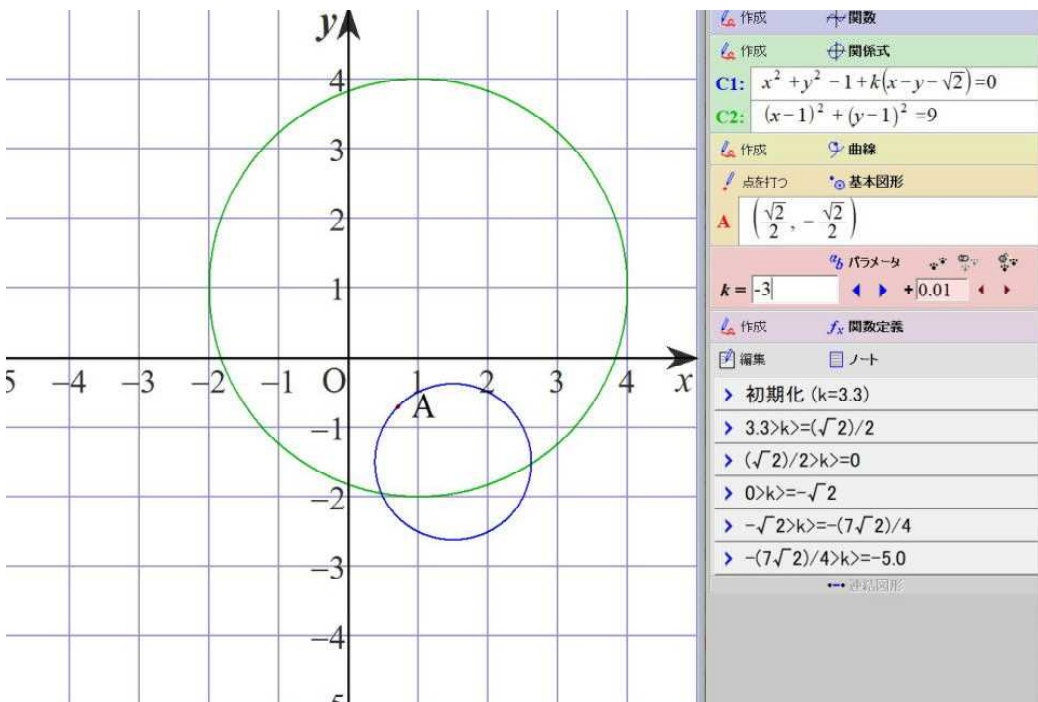
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(2) Experimental result (Grapes version simulation)

⑧ When the value of  $k$  is  $-7\sqrt{2}/4$



⑨ When the value of  $k$  is  $-3$



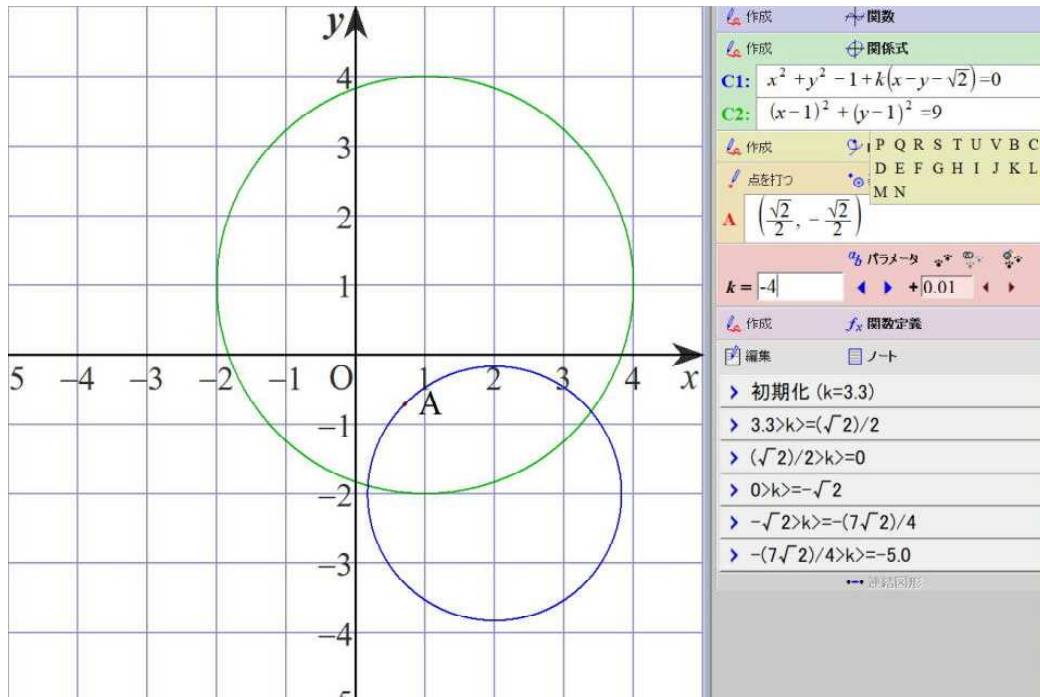
# Interesting Simulation III (Grapes)

1.24.2024  
Sohun

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(2) Experimental result (Grapes version simulation)

⑩ When the value of k is -4



⑪ When the value of k is -5

